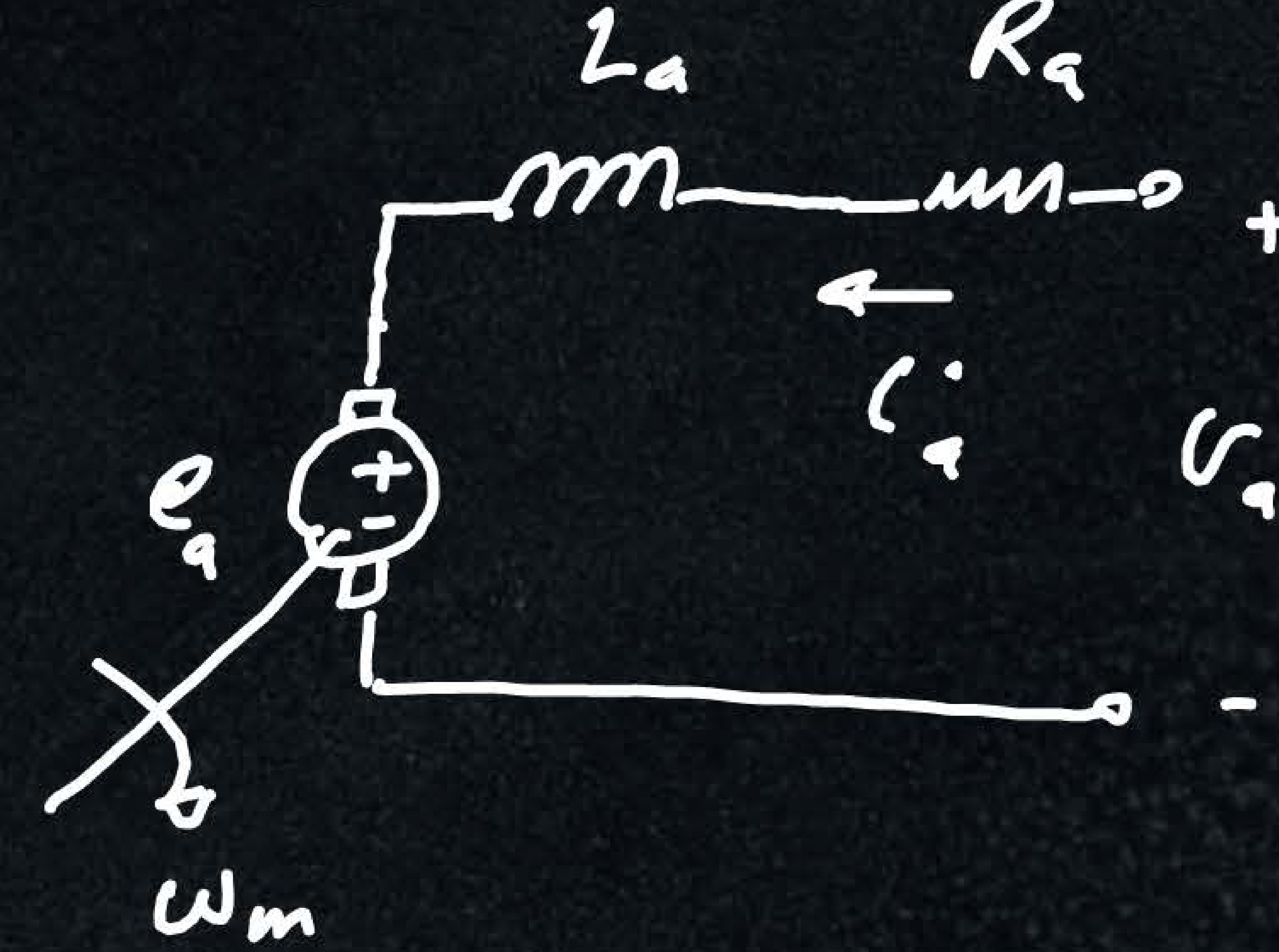
Block diagram and transfer function

$$\left.\begin{array}{l} V_a = R_a i_a + L_a \dfrac{di_a}{dt} + e_a \\[2ex] e_a = k\phi_f \omega_m = k' \omega_m \\[2ex] T_{el} = k\phi_f i_a = k' i_a \\[2ex] T_{el} = T_l + J \dfrac{d\omega_m}{dt} \end{array}\right]$$ t-domain



$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t) \circledast h(t)$$

$$Y(f) = \underbrace{H(s)}_{\text{Transfer Function}} X(f)$$

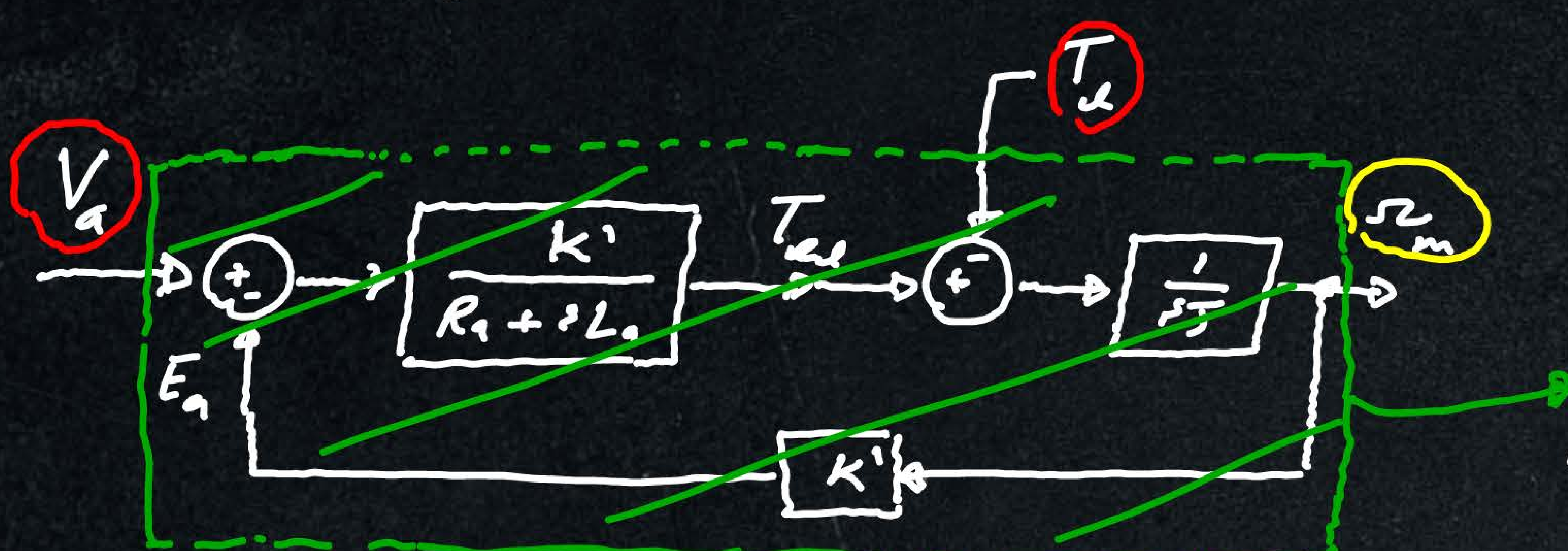
$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \xrightarrow{\mathcal{L}} V_a = (R_a + sL_a) I_a + E_a$$

$$e_a = k' \omega_m \xrightarrow{\mathcal{L}} E_a = k' \Omega_m$$

$$T_{el} = k' i_a \xrightarrow{\mathcal{L}} T_{el} = k' I_a$$

$$T_{el} = T_l + J \frac{d\omega_m}{dt} \xrightarrow{\mathcal{L}} T_{el} = T_l + sJ\Omega_m$$

s-domain



Block diagram of DC machine "constant flux"

Linear system

$$a_0 y + a_1 \frac{dy}{dt} + a_2 \frac{d^2 y}{dt^2} + \cdots + a_n \frac{d^{(n)} y}{dt^{(n)}} = b_0 x(t)$$

$y_1 \quad x_1$

$y_2 \quad x_2$

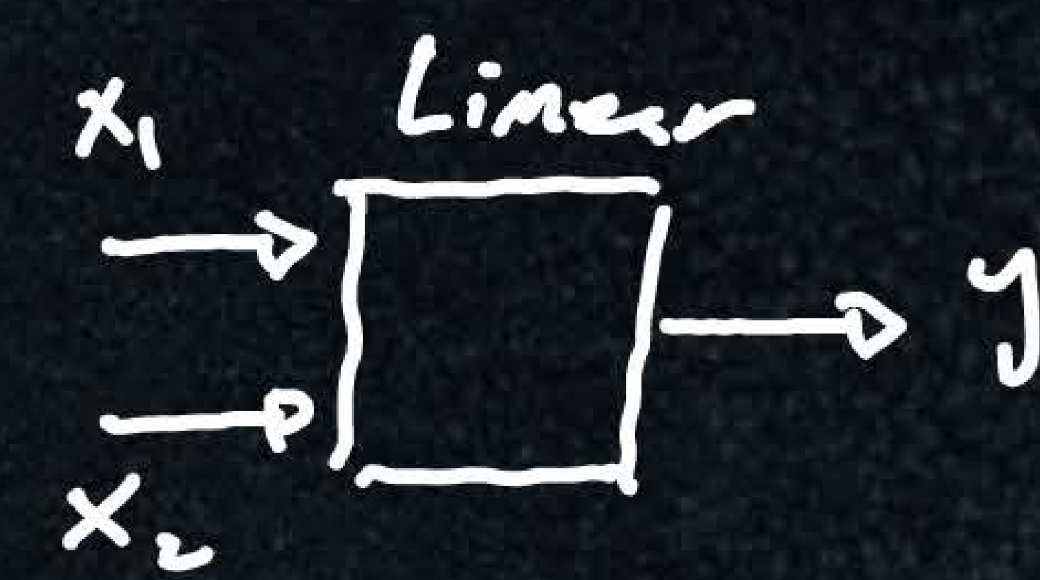
$y_3 \quad x_3$

$y_4 \quad x_4$

$y_5 \quad x_5$

$a_0, a_1, \cdots, a_n$ are constants.

x : input , y = output



$$\frac{Y(s)}{X(s)} = H(s) = \text{Transfer Function}$$

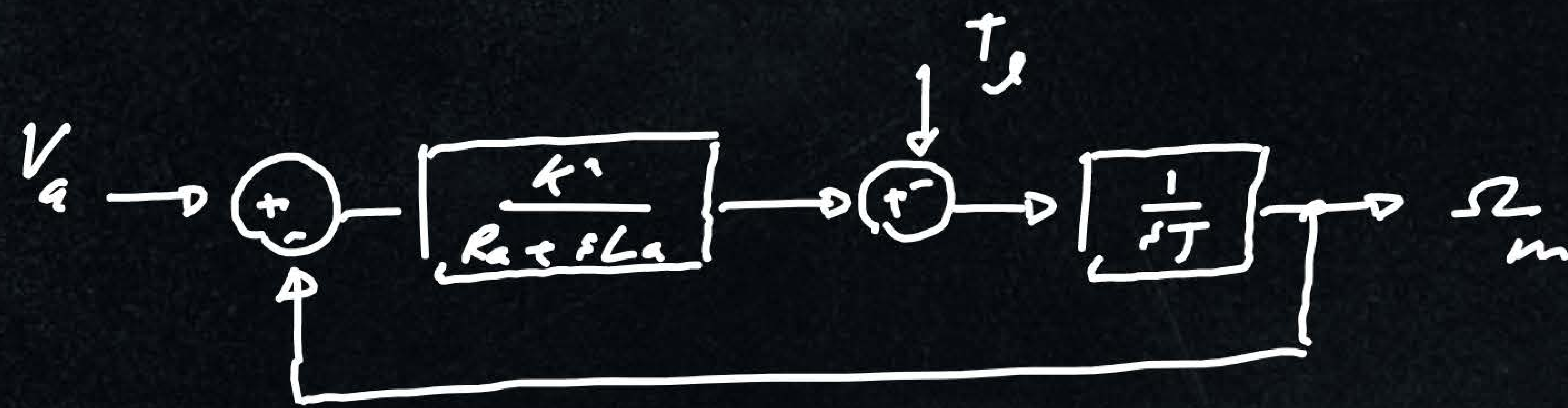
$y_1 \leftarrow x_1$

$y_2 \leftarrow x_2$

$y_1 + y_2 \leftarrow x_1 + x_2$

$$a_0 y + a_1 \frac{dy}{dt} = b_0 x$$

$$\left.\begin{aligned} a_0 y_1 + a_1 \frac{dy_1}{dt} &= b_0 x_1 \\ a_0 y_2 + a_1 \frac{dy_2}{dt} &= b_0 x_2 \end{aligned}\right] \rightarrow a_0 (y_1 + y_2) + a_1 \frac{d(y_1 + y_2)}{dt} = b_0 (x_1 + x_2)$$



$$\Omega_m = \Omega_{m_1} + \Omega_{m_2} \quad ; \quad \Omega_{m_1} = T_1 V_a \quad \text{when } T_l = 0$$

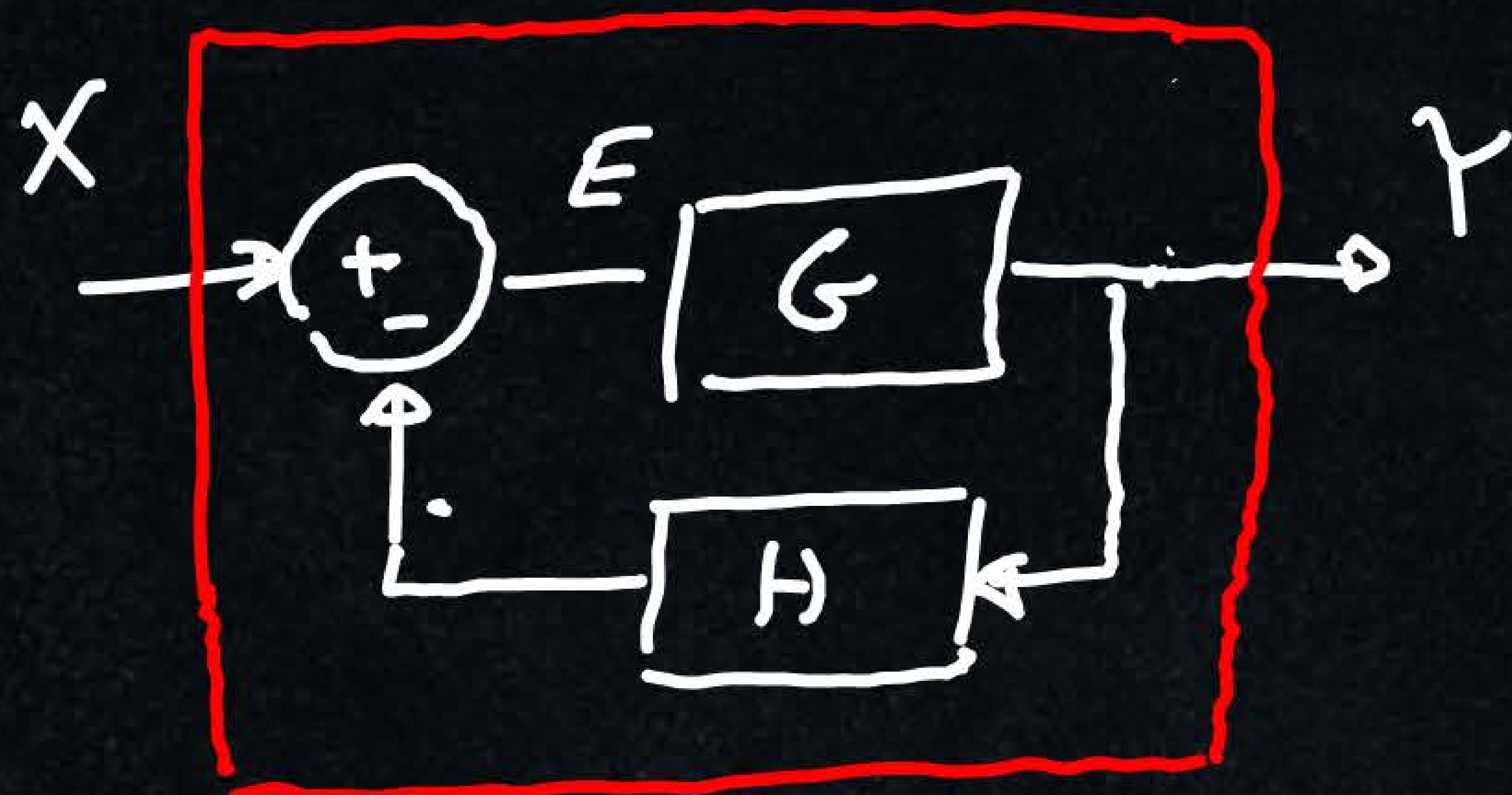
$$\Omega_{m_2} = T_2 T_l \quad \text{when } V_a = 0$$

$$\Omega_m = T_1 V_a + T_2 T_l .$$

where $$T_1 = \left. \frac{\Omega_m}{V_a} \right|_{T_l = 0}$$

$$T_2 = \left. \frac{\Omega_m}{T_l} \right|_{V_a = 0}$$

Closed loop system



$$\frac{Y}{X} = \frac{G}{1+GH}$$

G : Direct transfer function

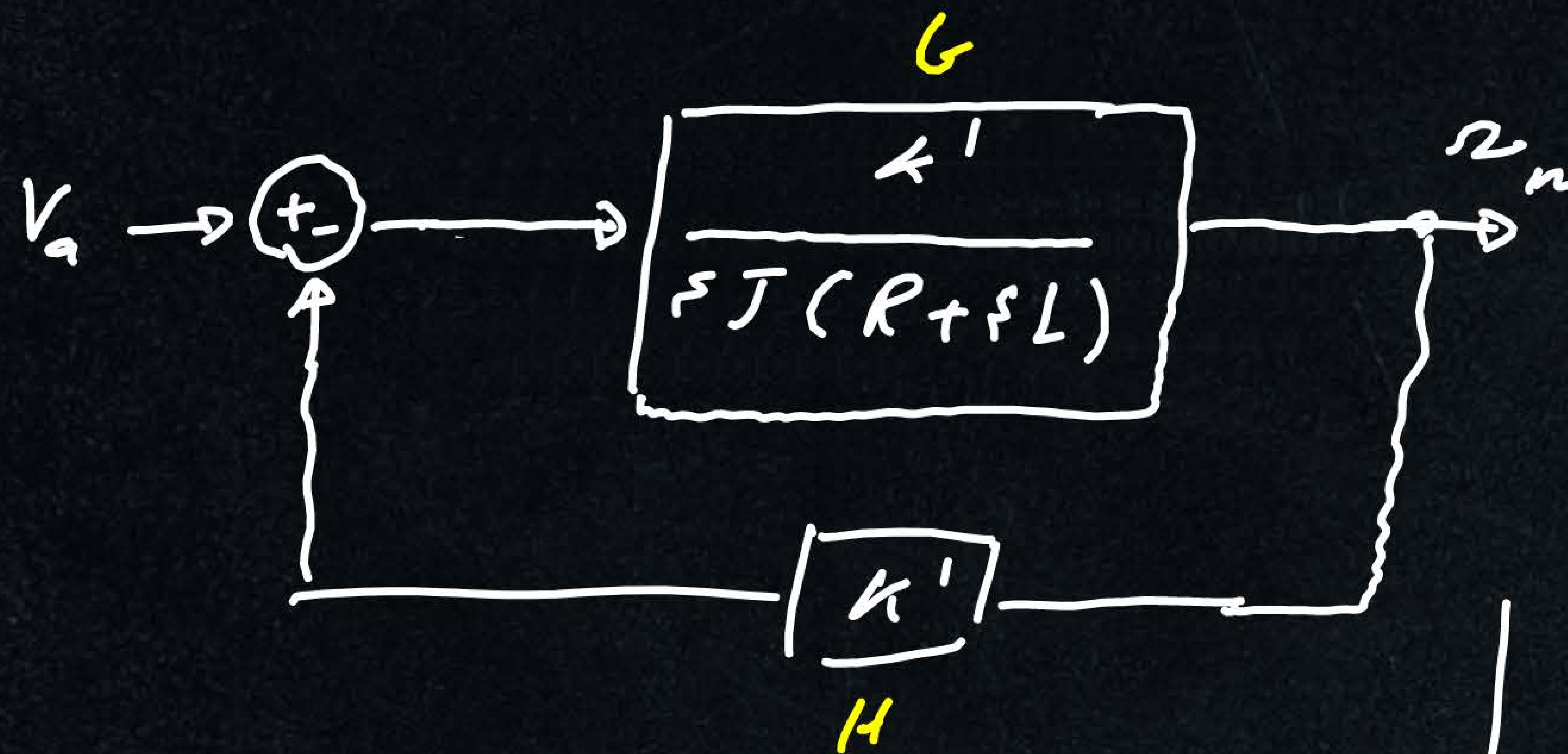
H : Feedback transfer function

$$Y = G\underline{E} = G(X - HY)$$

$$Y = GX - GHY \Rightarrow Y(1+GH) = GX$$

$$\frac{Y}{X} = \frac{G}{1+GH}.$$

$$T_1 = \Omega_m / V_a \quad \text{when} \quad T_l = 0$$



$$T_1 = \frac{G}{1+GH} \; ; \quad G = \frac{k'}{sJ(R+sL)}$$

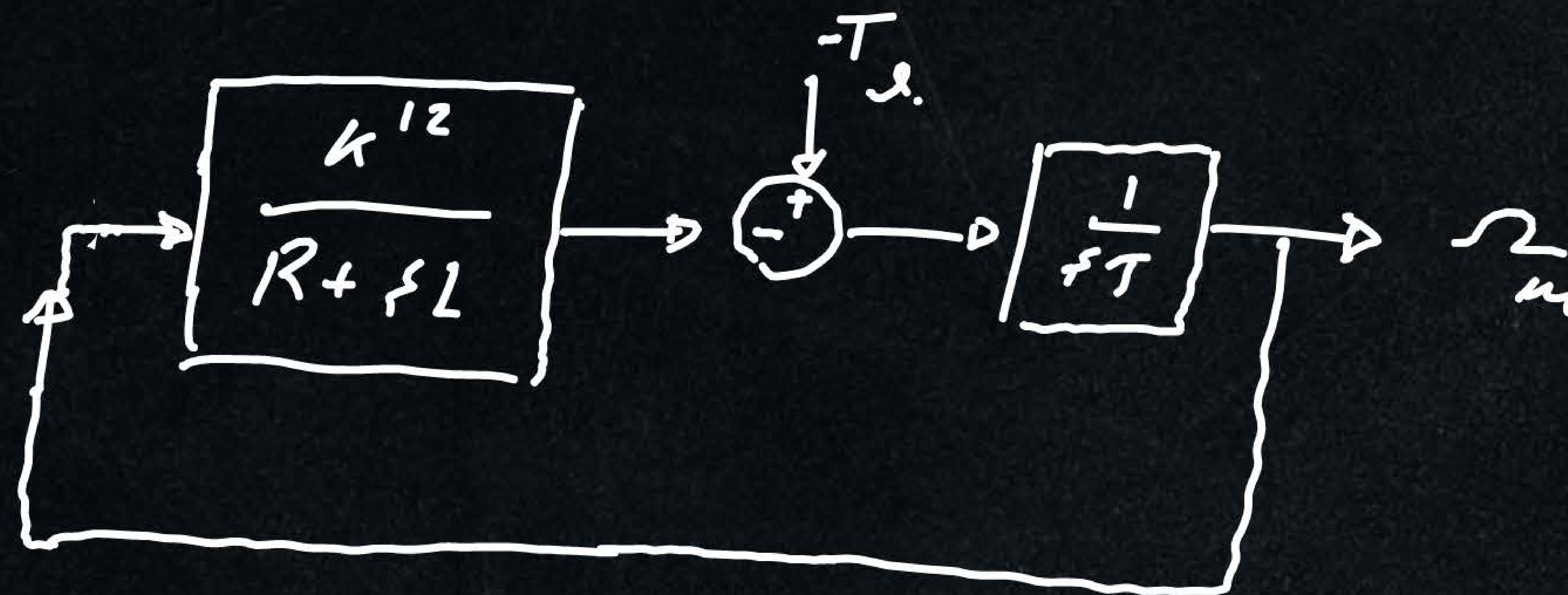
$$H = k'$$

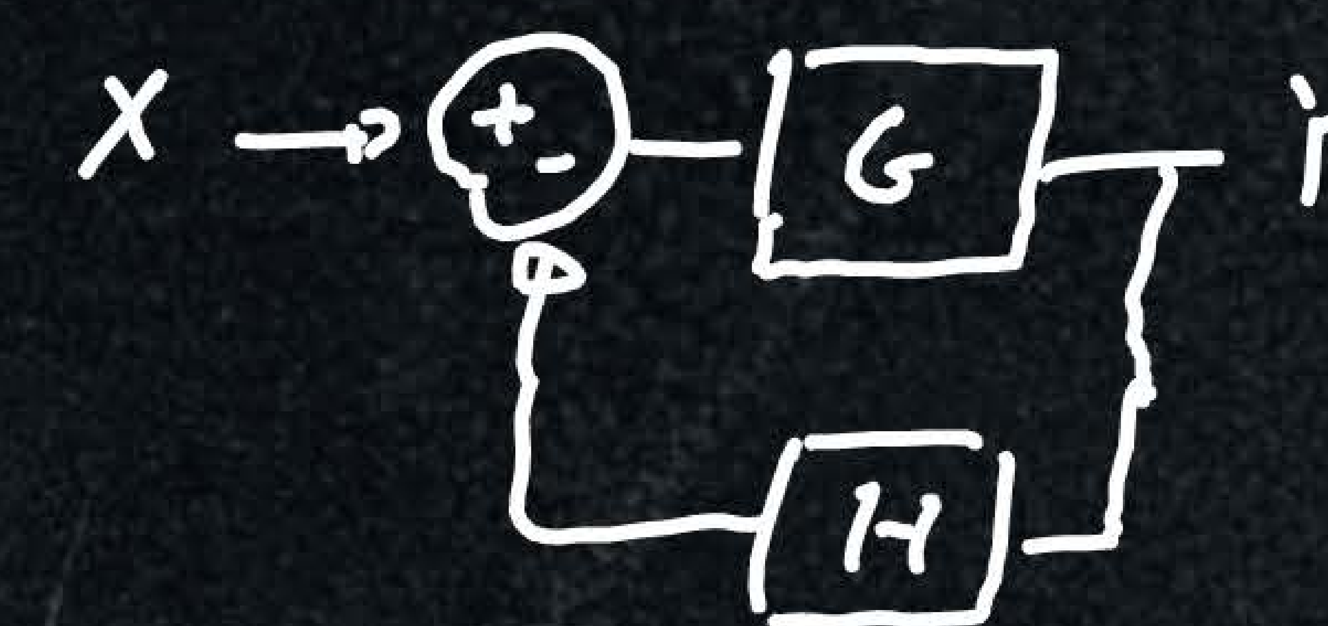
$$T_1 = \frac{\frac{k'}{sJ(R+sL)}}{1 + \frac{k'^2}{sJ(R+sL)}}$$

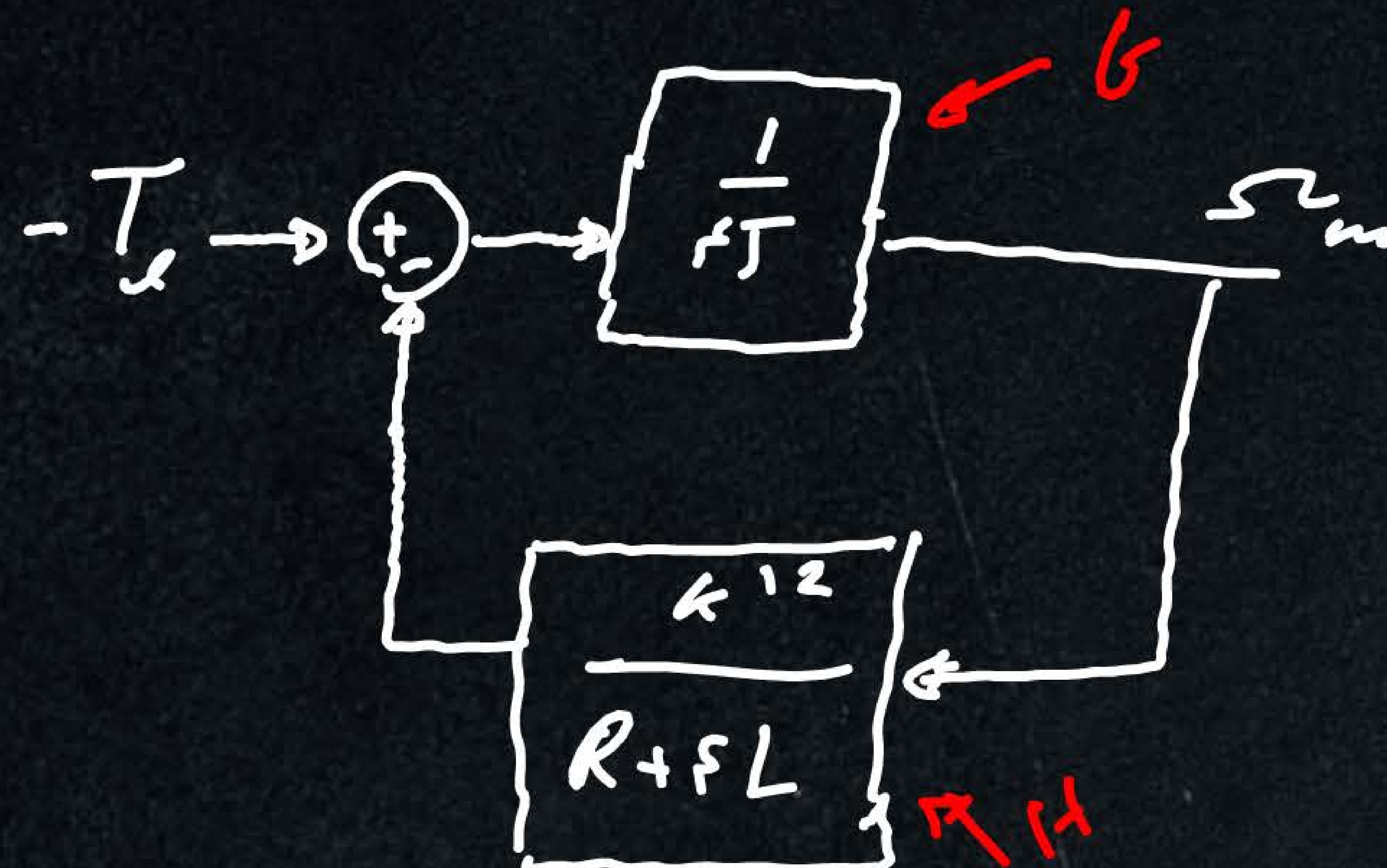
$$T_1 = \frac{k'}{sJ(R+sL) + k'^2}$$

$$T_1 = \frac{k'}{JLs^2 + JRs + k'^2}$$

$T_2 = \Omega_m / T_l$ when $V_a = 0$







$$T_2 = \frac{\Omega_m}{T_l} = \frac{-G}{1+GH}$$

$$G = \frac{1}{sJ} \quad ; \quad H = \frac{k'^2}{R+sL}$$

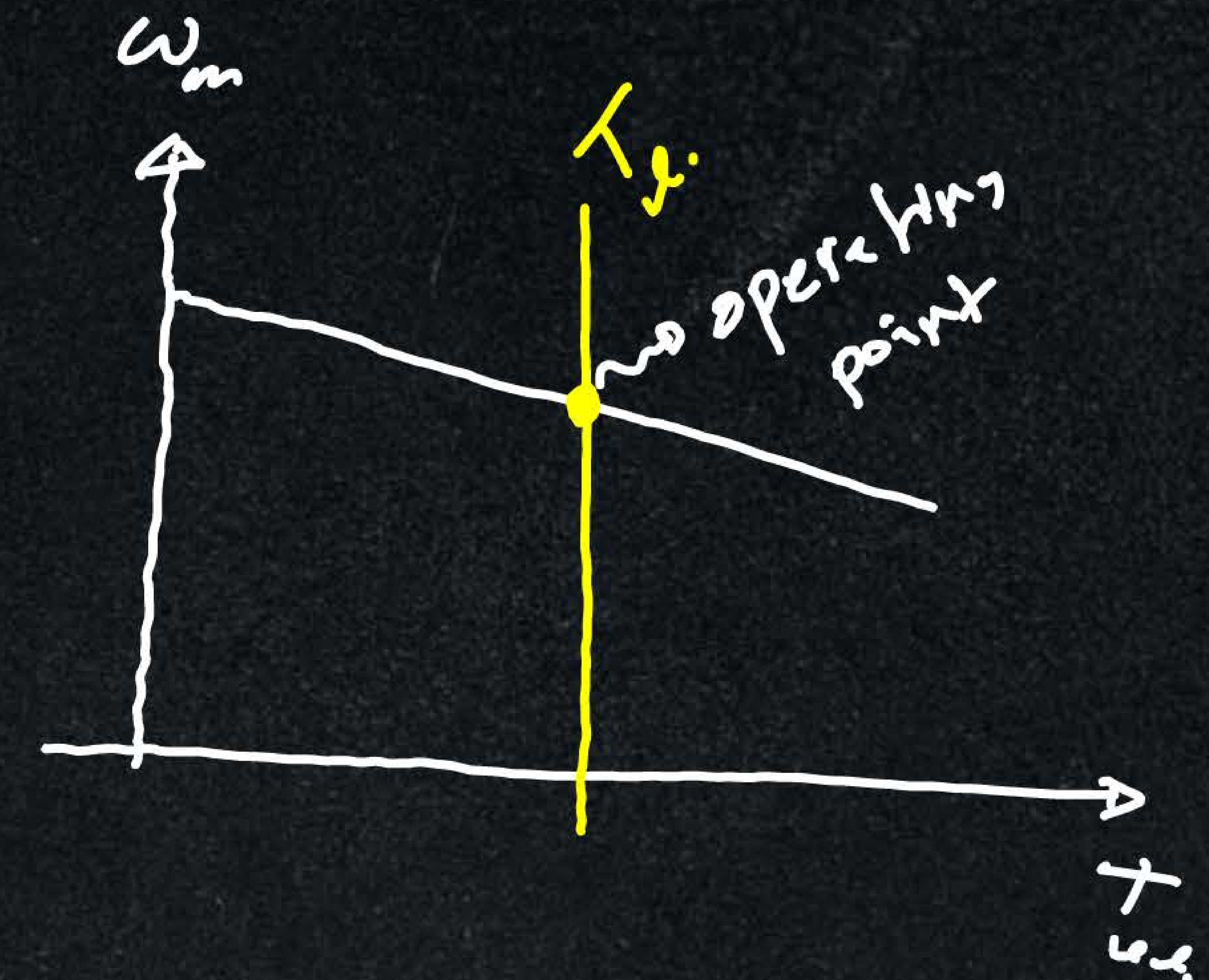
Separately + shunt excited DC motors

$$V_a = R_a i_a + e_a \; ; \; e_a = K\Phi_f \omega_m \Rightarrow V_a = R_a i_a + K\Phi_f \omega_m \quad \ldots (1)$$

$$T_{el} = K\Phi_f i_a \Rightarrow i_a = \frac{T_{el}}{K\Phi_f} \quad \ldots (2)$$

$$(2) \longrightarrow (1) \Rightarrow \quad V_a = \frac{R_a}{K\Phi_f} T_{el} + K\Phi_f \omega_m$$

$$\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}$$



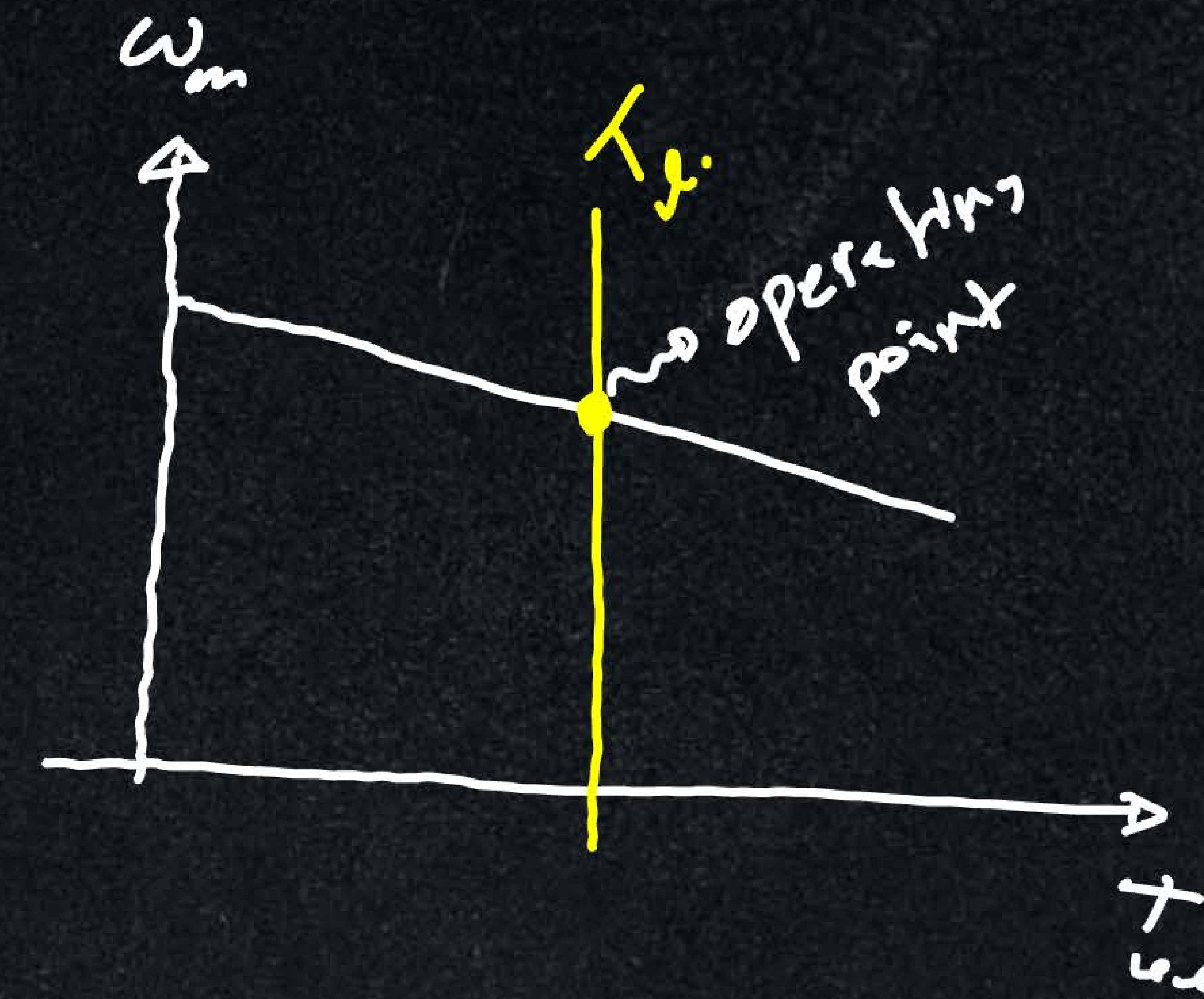
Separately + shunt excited DC motors

$$V_a = R_a i_a + e_a \; ; \; e_a = K\Phi_f \omega_m \Rightarrow V_a = R_a i_a + K\Phi_f \omega_m \quad \ldots (1)$$

$$T_{el} = K\Phi_f i_a \Rightarrow i_a = \frac{T_{el}}{K\Phi_f} \quad \ldots (2)$$

$$(2) \rightarrow (1) \Rightarrow \quad V_a = \frac{R_a}{K\Phi_f} T_{el} + K\Phi_f \omega_m$$

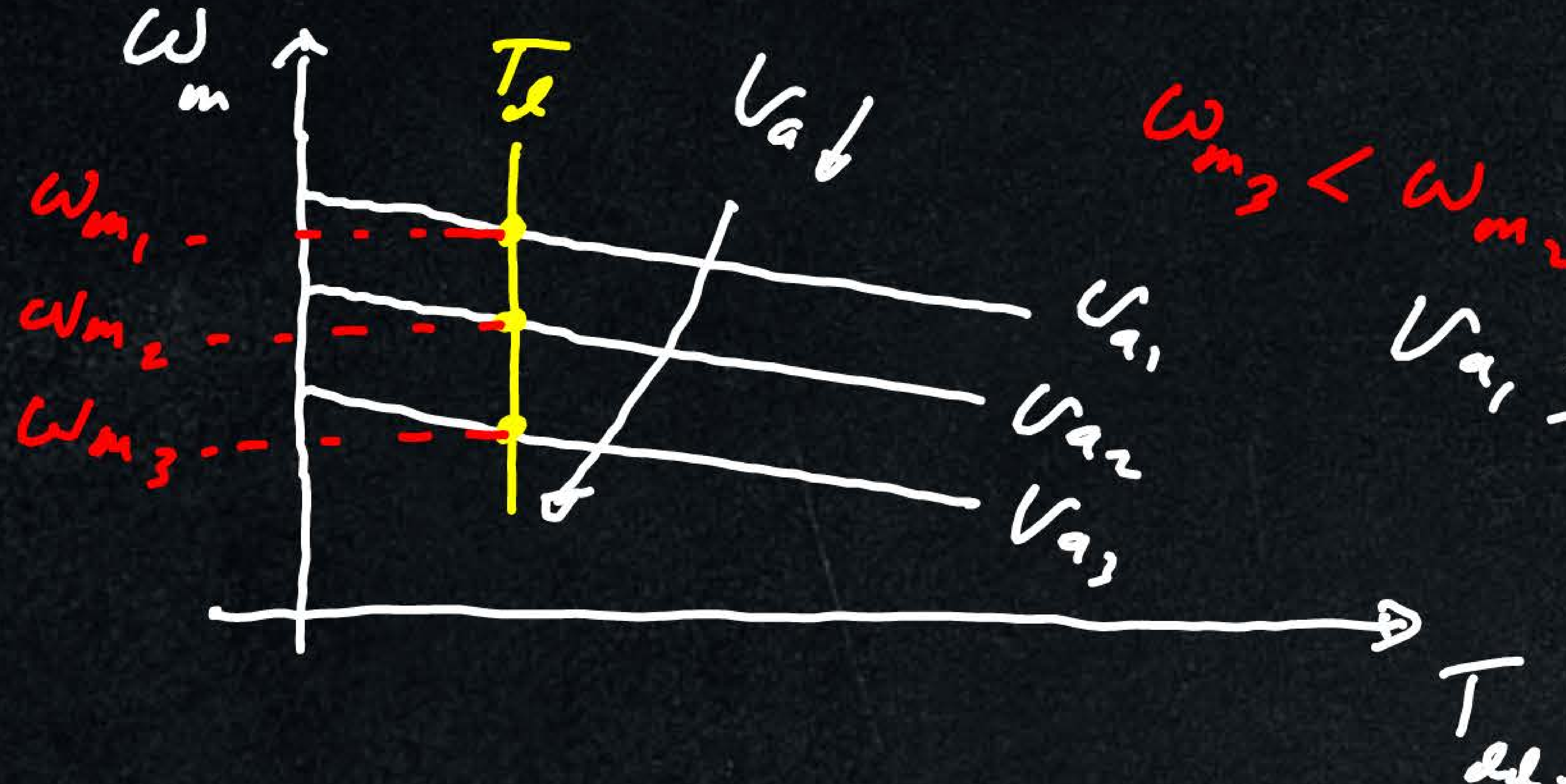
$$\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}$$



Methods of speed control

$$\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}$$

- V_a control

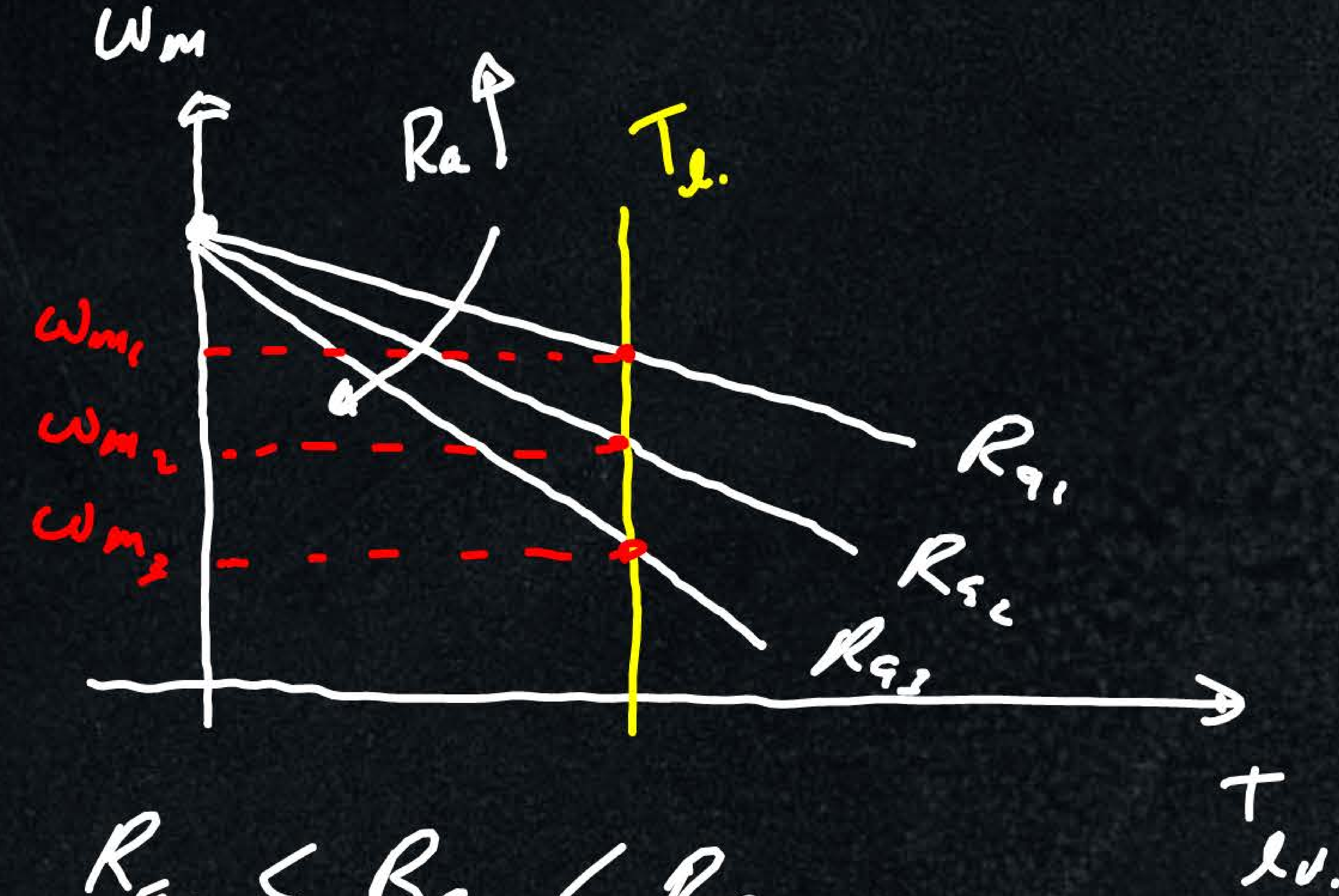


$\omega_{m_3} < \omega_{m_2} < \omega_{m_1}$

$V_{a_1} > V_{a_2} > V_{a_3}$

• R_a control

$$\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}$$



$R_{a1} < R_{a2} < R_{a3}$

$\omega_{m1} > \omega_{m2} > \omega_{m3}$

$R_a \uparrow \Rightarrow \omega_m \downarrow$

Torque-speed equation "shunt & separately excited DC motors"

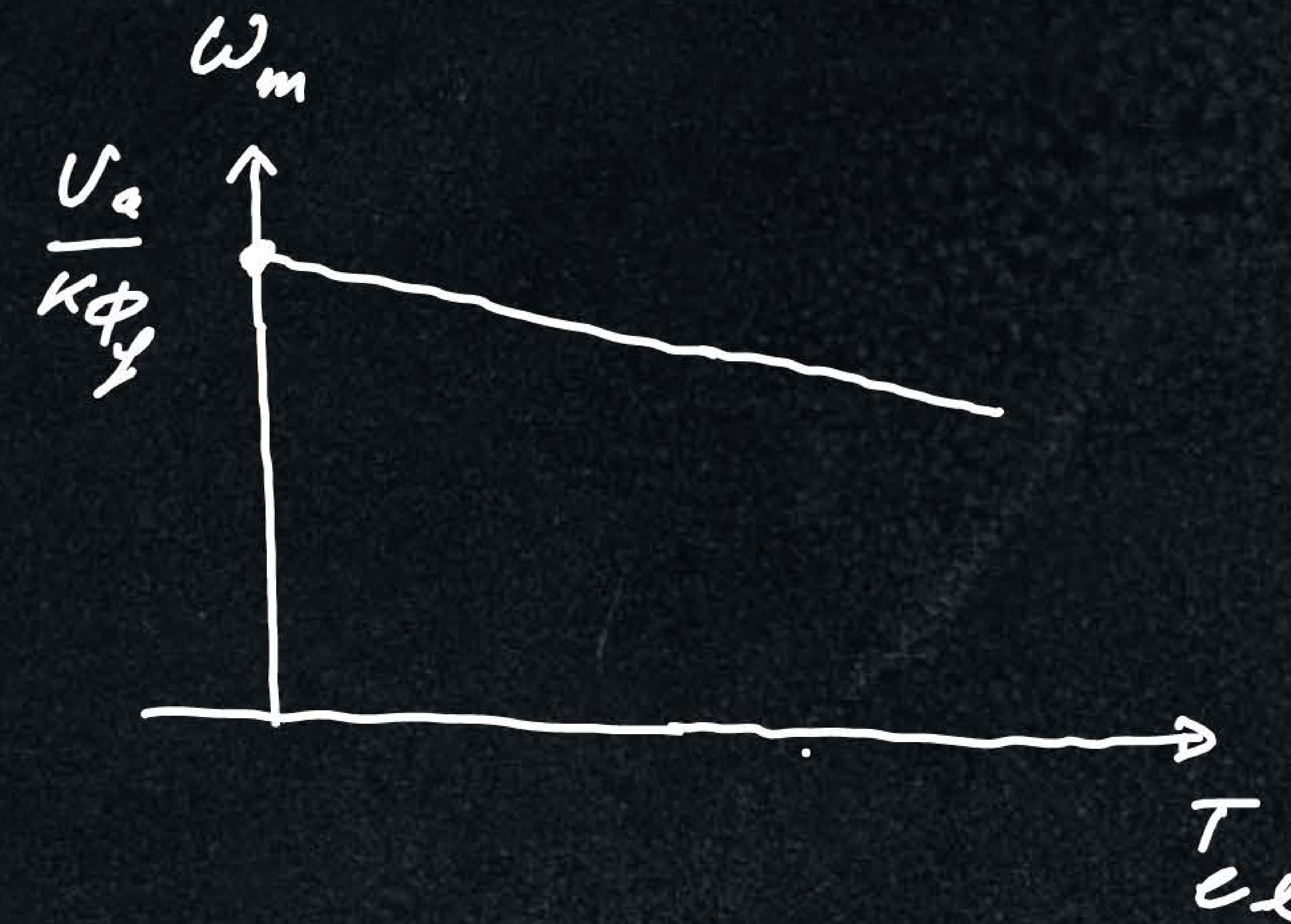
$$V_a = R_a i_a + e_a \quad ; \quad e_a = K\Phi_f \omega_m$$

$$V_a = R_a i_a + K\Phi_f \omega_m \quad \ldots (1)$$

$$T_{el} = K\Phi_f i_a \Rightarrow i_a = \frac{T_{el}}{K\Phi_f} \quad \ldots (2)$$

$$(2) \rightarrow (1) \Rightarrow V_a = \frac{R_a}{K\Phi_f} T_{el} + K\Phi_f \omega_m$$

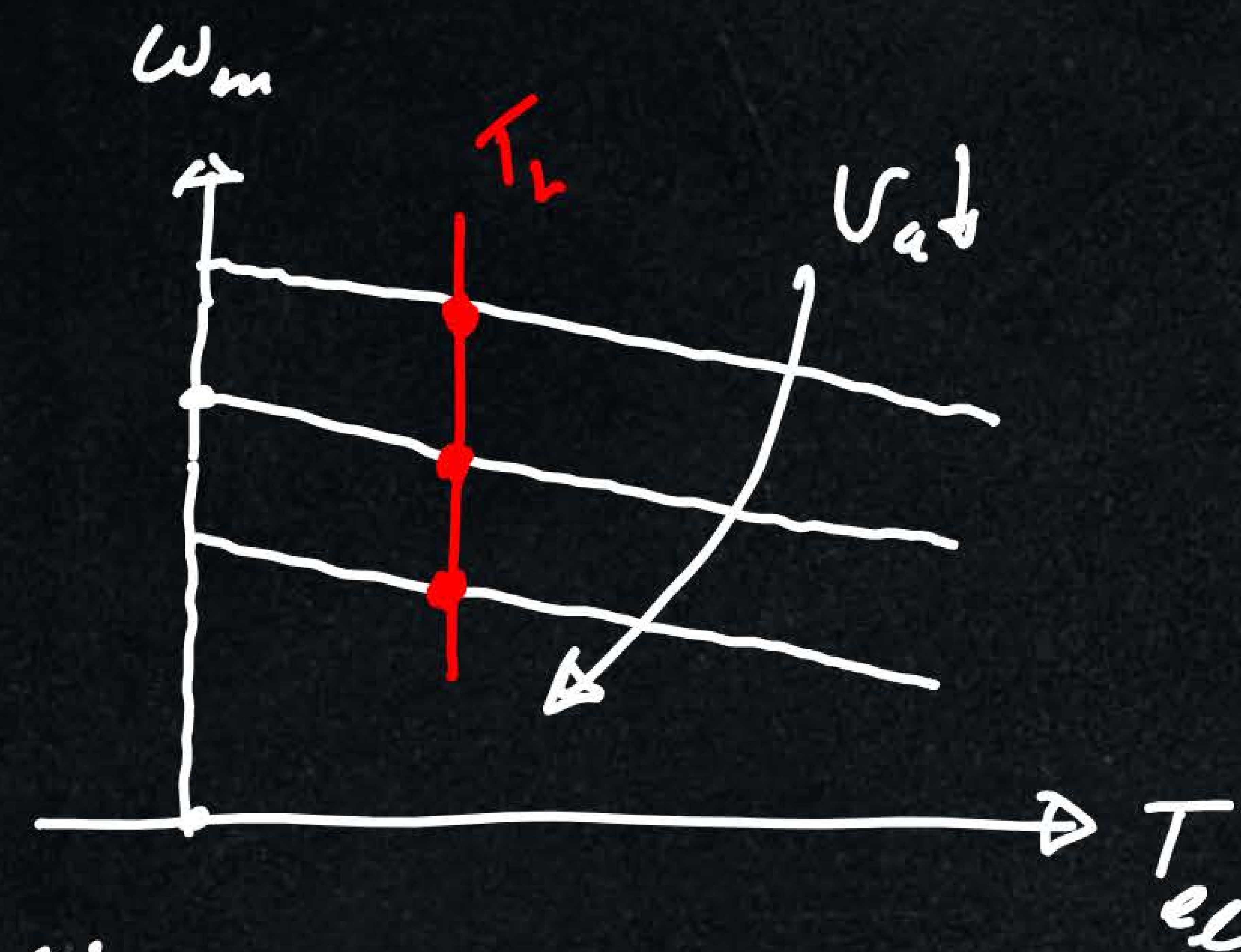
$$\boxed{\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}}$$



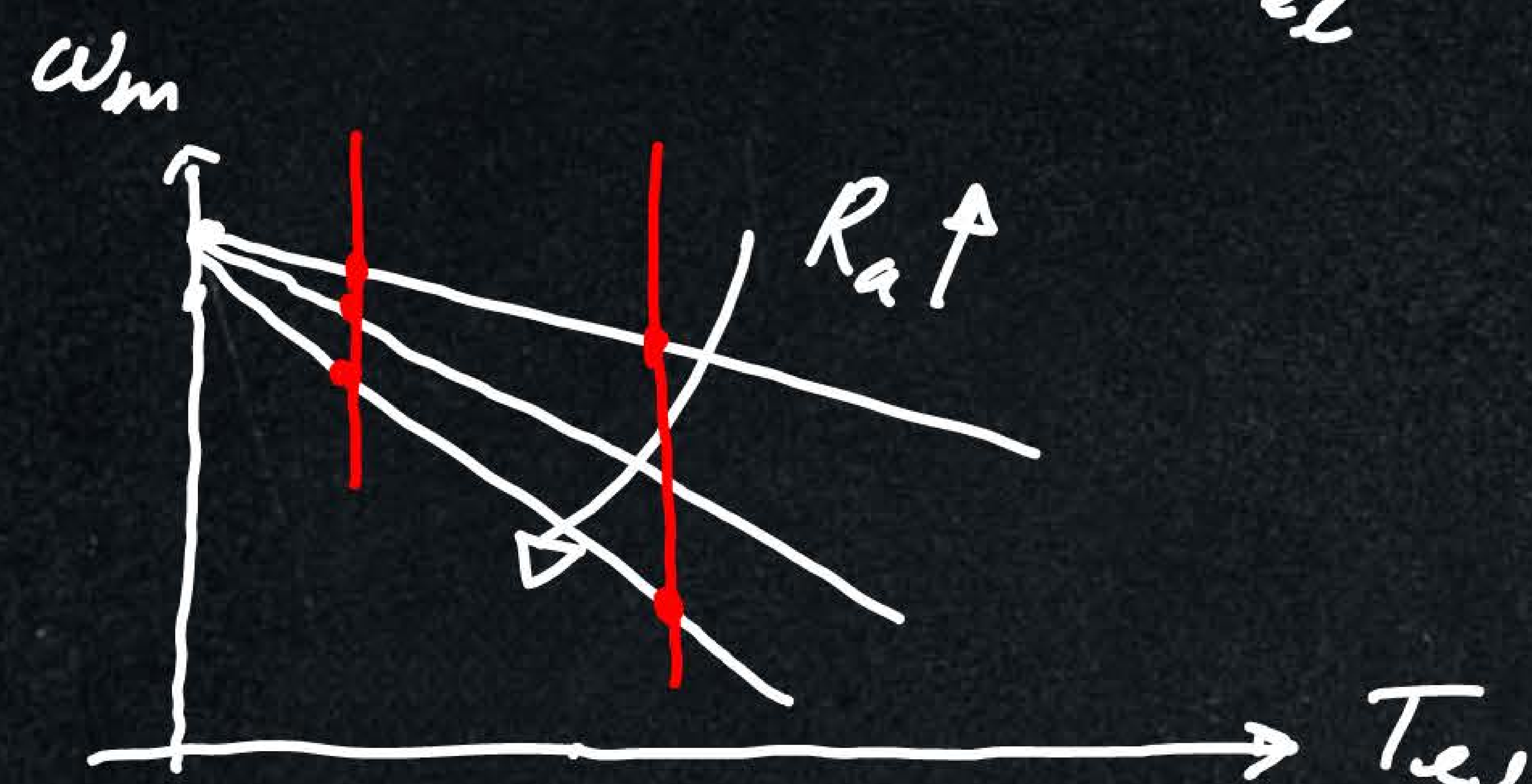
$$\omega_m = \frac{V_a}{K\Phi_f} - \frac{R_a}{(K\Phi_f)^2} T_{el}.$$

Methods of speed control :-

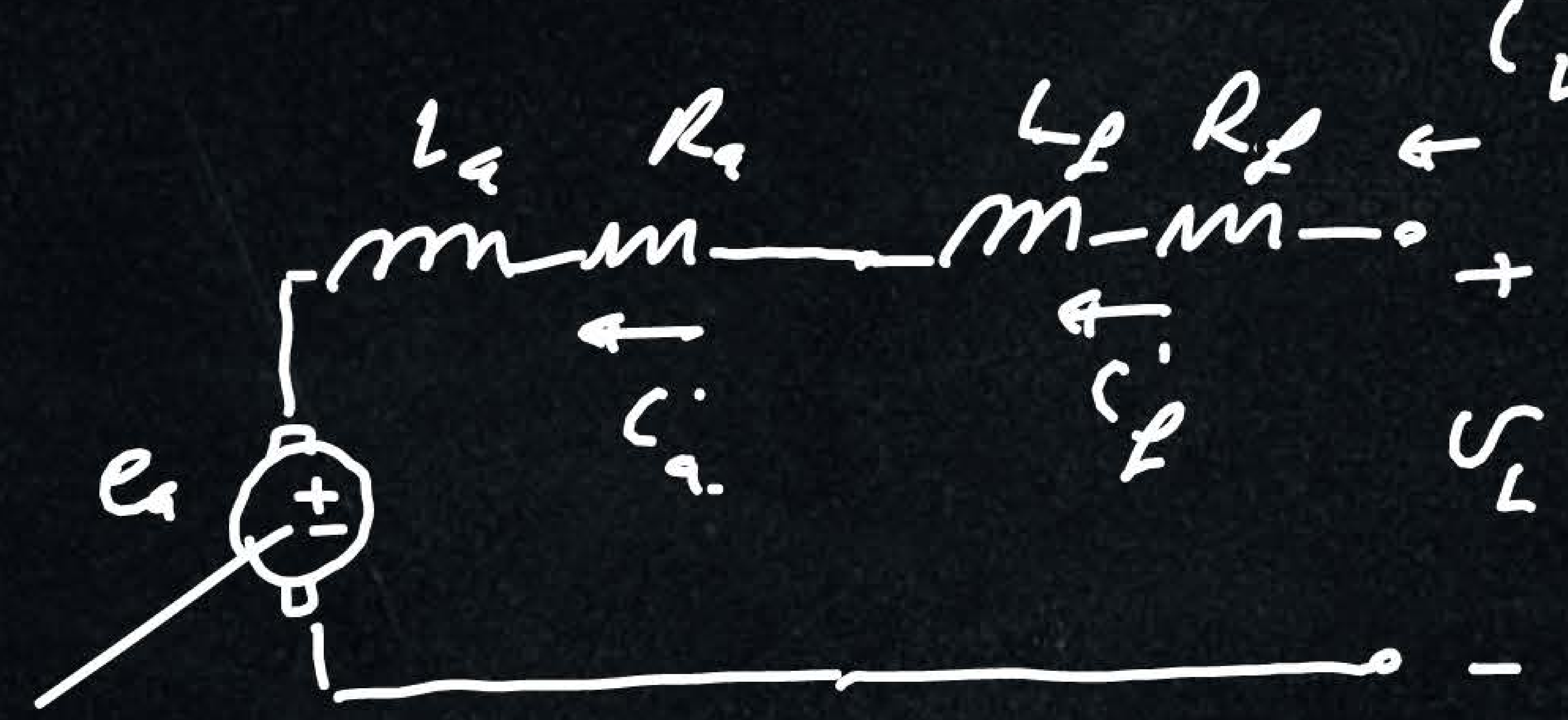
1) V_a control.



2) R_a control



Torque-speed equation (series DC motor)



$$V_L = (R_f + R_a)\, i_a + K\phi_f\, \omega_m$$

$$\phi_f = C\, i_f$$

$$i_f = i_a$$

$$\phi_f = C\, i_a$$

$$\Rightarrow V_L = (R_f + R_a)\, i_a + KC\, i_a\, \omega_m \quad \cdots (1)$$

$$T_{el} = K\phi_f\, i_a = KC\, i_a^2 \Rightarrow i_a = \sqrt{\frac{T_{el}}{KC}} \quad \cdots (2)$$

(2) → (1) ⇒

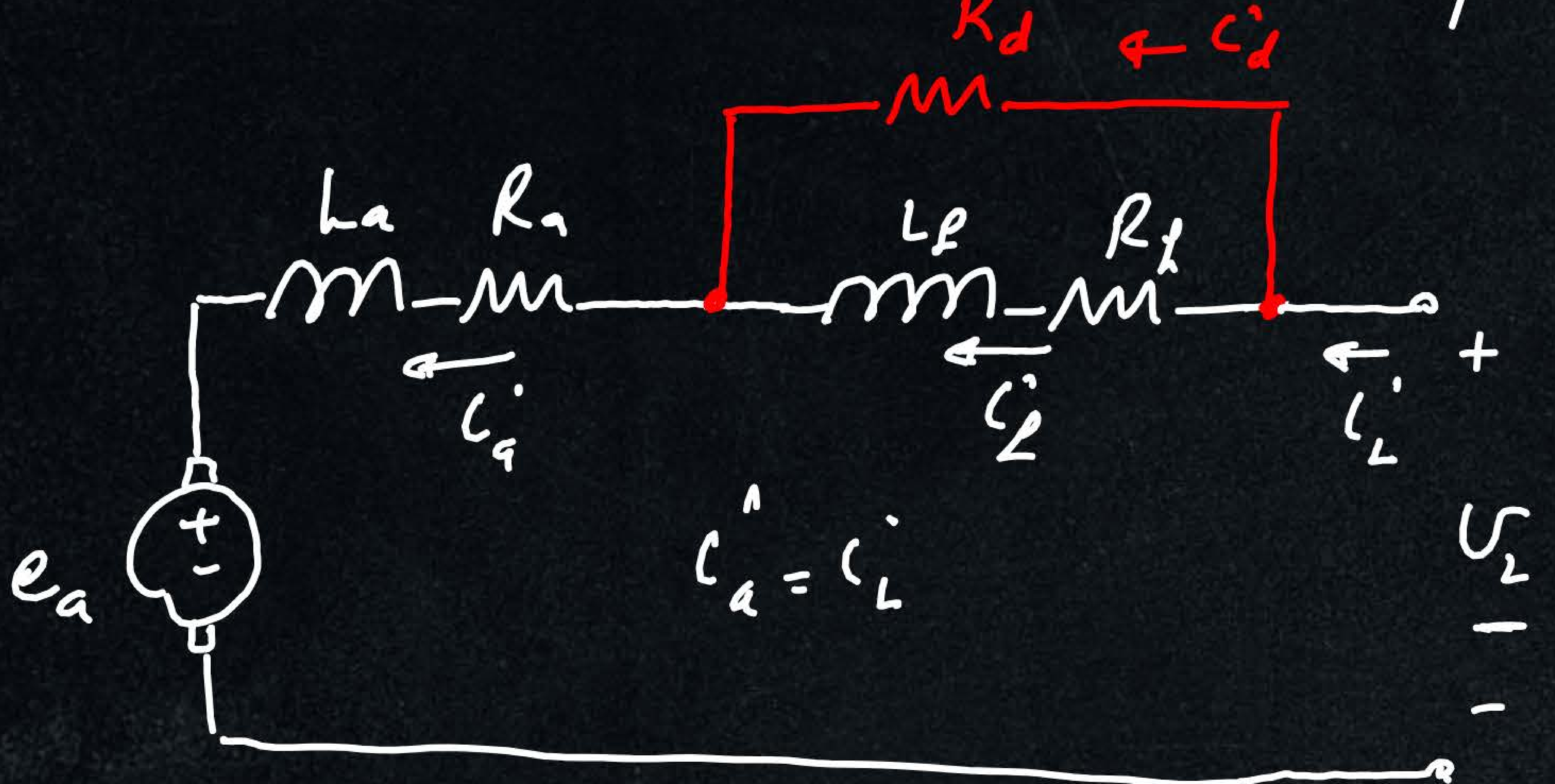
$$V_L = \frac{(R_f + R_a)}{\sqrt{KC}}\sqrt{T_{el}} + \sqrt{KC}\,\sqrt{T_{el}}\,\omega_m$$

$$\left\{ \omega_m = \frac{V_L}{\sqrt{KC}}\,\frac{1}{\sqrt{T_{el}}} - \frac{(R_f + R_a)}{KC} \right\}$$

$$\omega_m = \frac{V_t}{\sqrt{KC}} \frac{1}{\sqrt{T_{el}}} - \frac{R_a + R_f}{KC}$$

Field control "series DC motor"

(Field diverting resistor method)



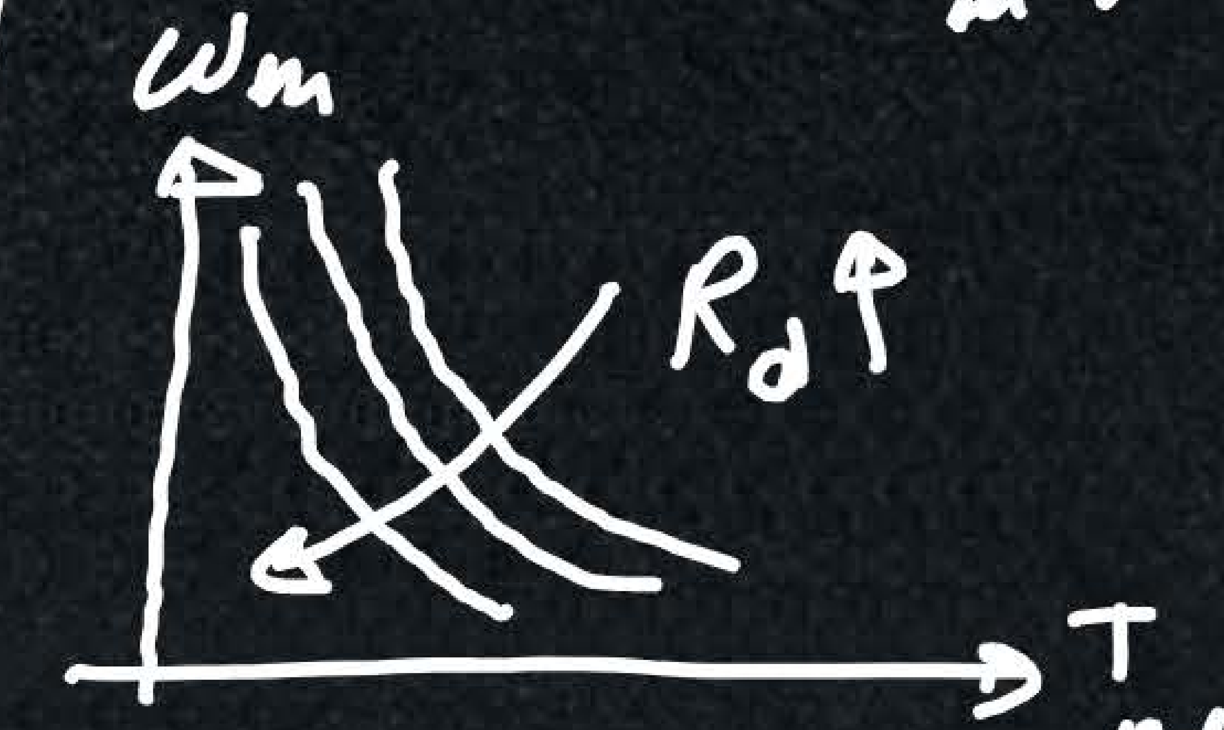
$$V_L = R_a i_L + \frac{R_f R_d}{R_f + R_d} i_L + K\phi_f \omega_m$$

$$\phi_f = C i_f = C \frac{R_d}{R_d + R_f} i_L$$

$$V_L = R_a i_L + \frac{R_f R_d}{R_f + R_d} i_L + KC \frac{R_d}{R_d + R_f} i_L \omega_m$$

$R_d \uparrow \Rightarrow i_f \uparrow$

$\Rightarrow \Phi \uparrow \Rightarrow \omega_m \downarrow$



EX :- A separately excited DC motor used to drive a fan whose torque is proportional to ω_m^2. When the armature circuit is connected across 200 V, it takes armature current of 16 A and the motor runs at speed of 1000 rpm. If the speed of the motor is to be reduced to 750 rpm, calculate the required voltage and the current drawn by the motor. ($R_a = 0.5\ \Omega$).

$$V_{a_1} = R_a i_{a_1} + e_{a_1}$$

$$200 = 0.5(16) + e_{a_1} \Rightarrow e_{a_1} = 192\ V$$

$$\frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{n_1} = \frac{750}{1000} = \frac{e_{a_2}}{192}$$

$$\Rightarrow e_{a_2} = 144.36\ V$$

$$T_{el} = K\Phi_f i_a = T_L = A\,\omega_m^2$$

$$\frac{T_{el_1}}{T_{el_2}} = \frac{\cancel{K\Phi_f}\, i_{a_1}}{\cancel{K\Phi_f}\, i_{a_2}} = \frac{\cancel{A}\,\omega_{m_1}^2}{\cancel{A}\,\omega_{m_2}^2}$$

$$\frac{i_{a_1}}{i_{a_2}} = \frac{\omega_{m_1}^2}{\omega_{m_2}^2} = \left(\frac{1000}{750}\right)^2 = \frac{16}{i_{a_2}} \Rightarrow \boxed{i_{a_2} = 9\ A}$$

$$V_{a_2} = 0.5(9) + 144.36 = \boxed{150\ \checkmark}$$

solution

$$V_{a_1} = R_a i_{a_1} + e_{a_1}$$

$$200 = 0.5(16) + e_{a_1} \Rightarrow e_{a_1} = 192\ \checkmark$$

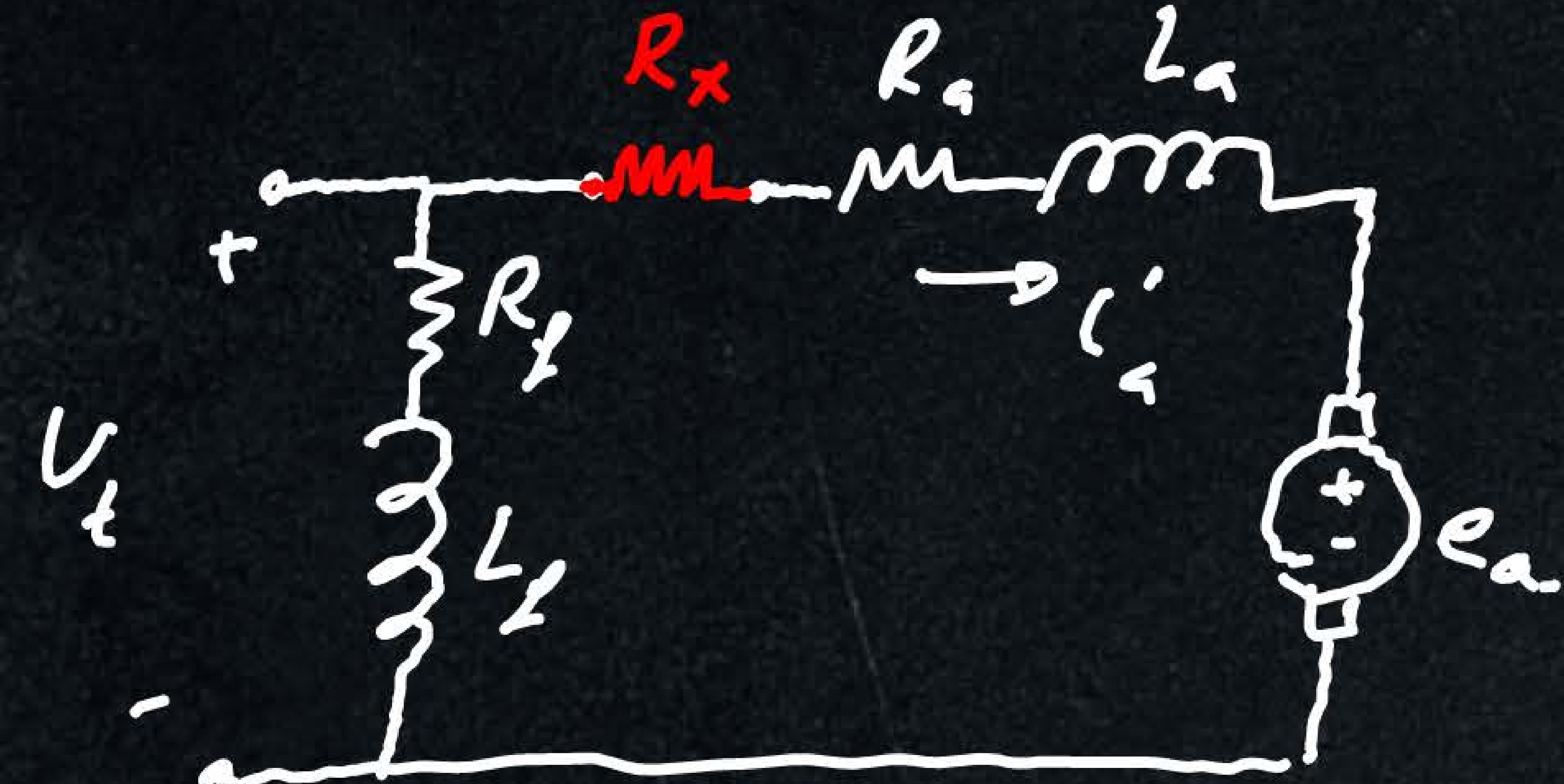
$$T \propto x^2$$

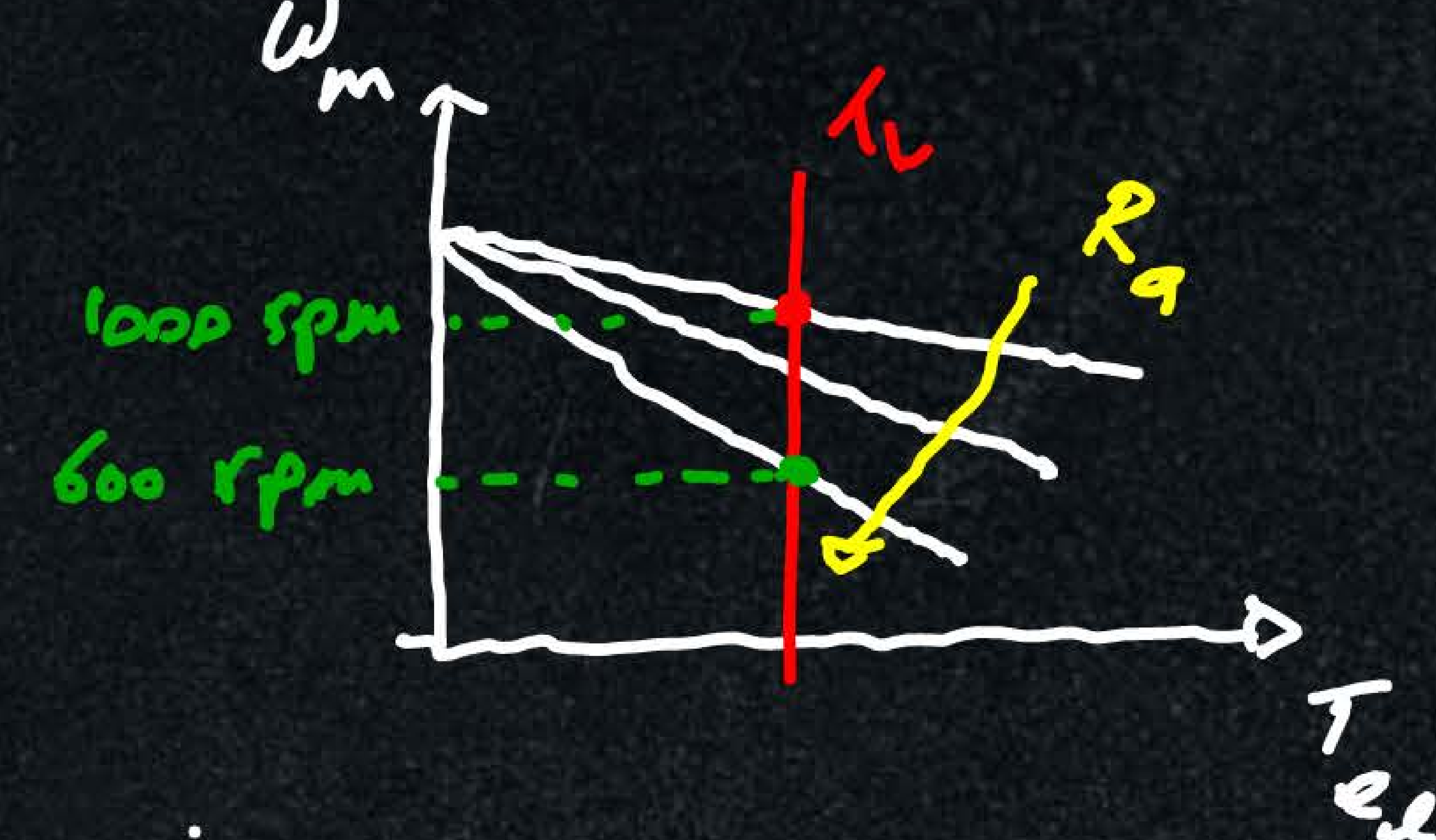
$$\frac{T_1}{T_2} = \frac{x_1^2}{x_2^2}$$

$$\frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{n_1} = \frac{750}{1000} = \frac{e_{a_2}}{192}$$

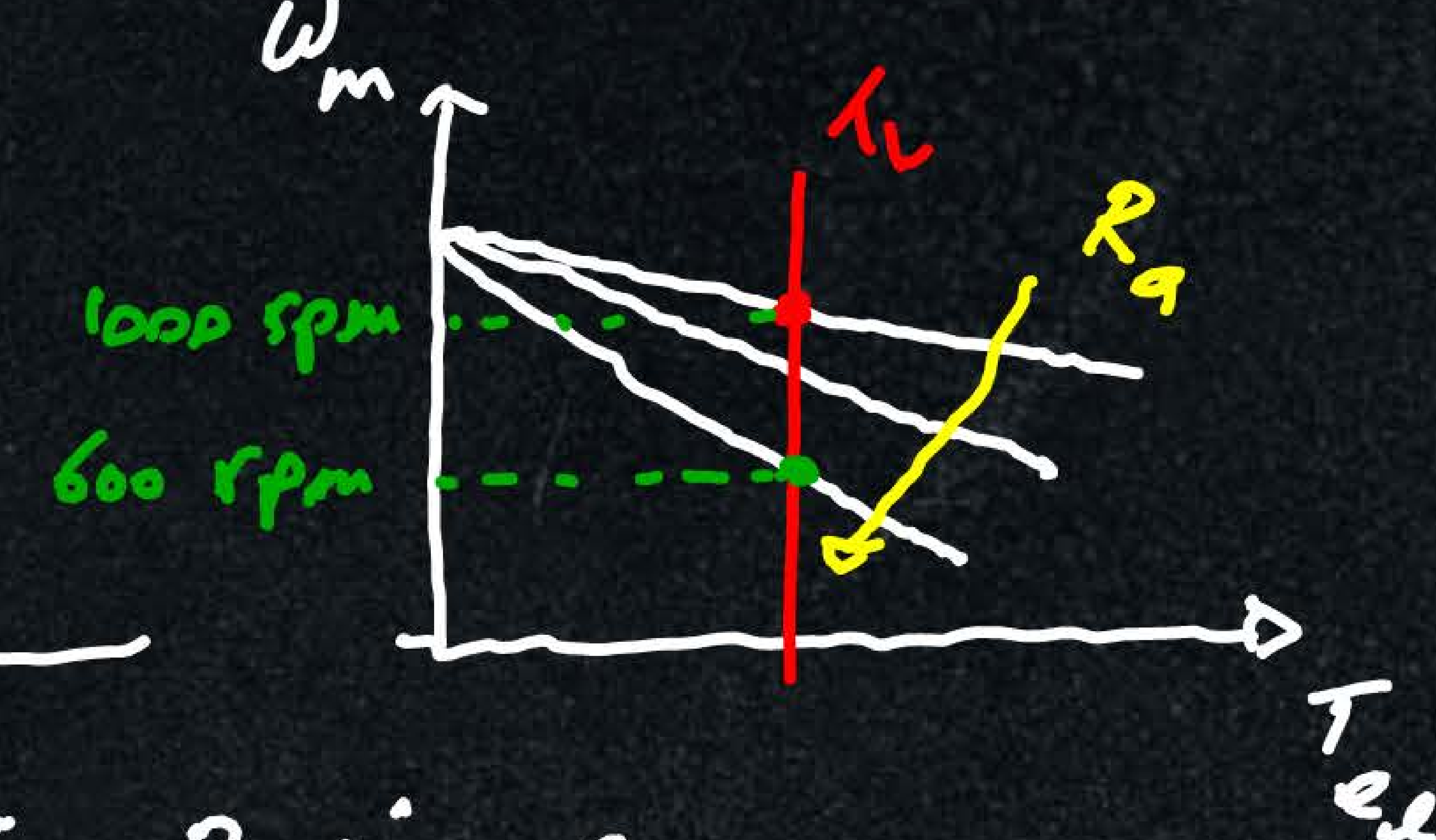
$$e_{a_2} = 144.36\ \checkmark$$

EX :- A 240 V DC shunt motor has an armature resistance of 0.2 Ω. When the armature current is 40 A, the speed is 1000 rpm. (a) Find additional resistance, R_x, to be connected in series with armature to reduce the speed to 600 rpm. Assume the armature current remains the same.





EX :- A 240 V DC shunt motor has an armature resistance of 0.2 Ω. When the armature current is 40 A, the speed is 1000 rpm. (a) Find additional resistance, R_x, to be connected in series with armature to reduce the speed to 600 rpm. Assume the armature current remains the same.



$$V_a = R_a i_a + e_{a_1}$$

$$240\,V = 0.2(40) + e_{a_1}$$

$$e_{a_1} = 240 - 8 \Rightarrow \boxed{e_{a_1} = 232\,V}$$

$$V_a = (R_a + R_x)\, i_a + e_{a_2}$$

$$240 = (0.2 + R_x)\,40 + e_{a_2}$$

$$\boxed{e_{a_2} = 232 - 40 R_x}$$

$$\left.\begin{aligned} e_{a_1} &= 232 \text{ V} \\ e_{a_2} &= 232 - 40 R_x \end{aligned}\right\} \Rightarrow \frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{n_1} = \frac{600}{1000} = \frac{232 - 40R_x}{232}$$

solve for $R_x \Rightarrow \boxed{R_x = 2.325\ \Omega}$

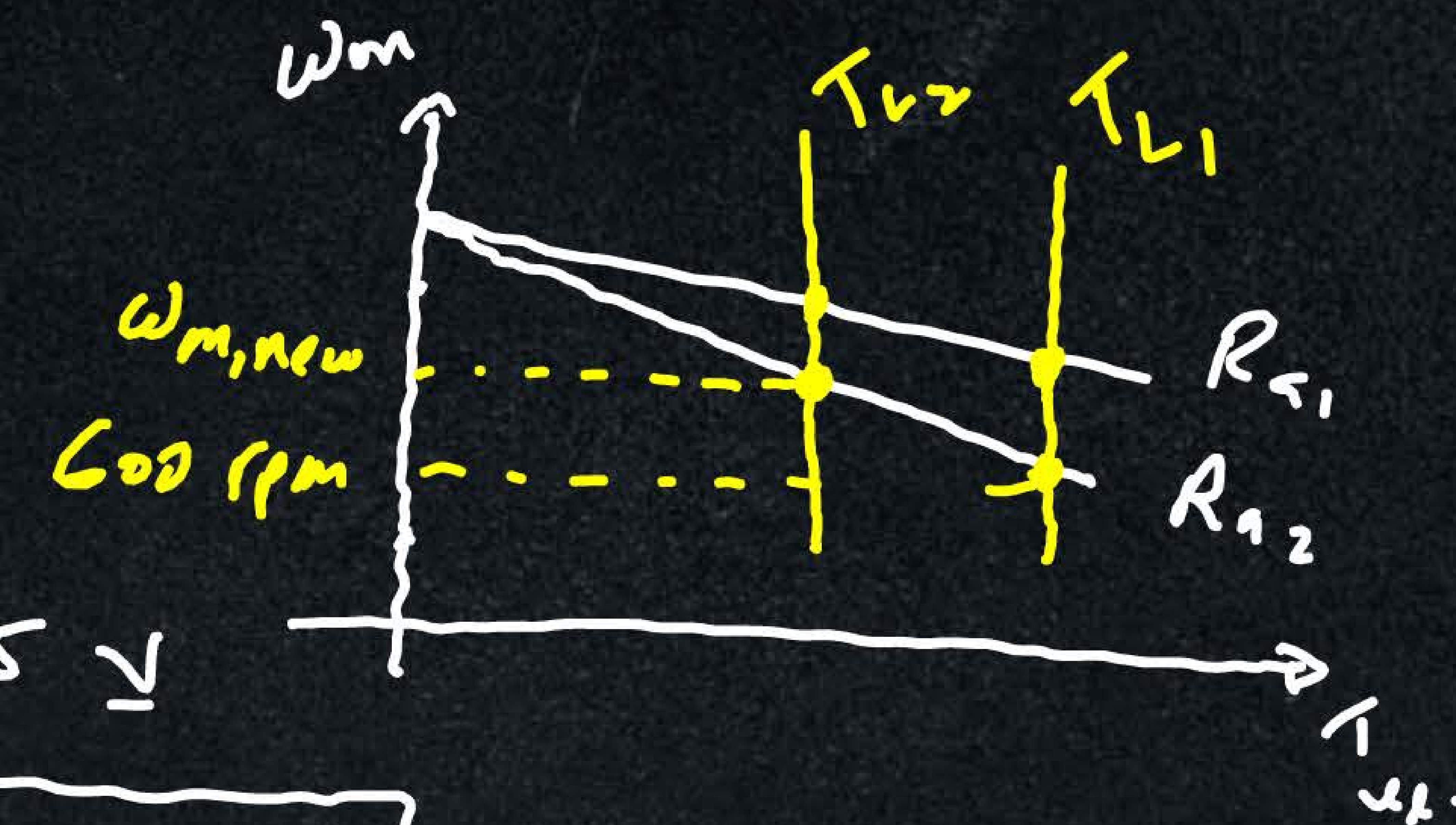
(b) If the current decreases to 20 A (with resistance R_x connected) find the new speed of the motor.

$e_{a_1} = 232$ V

$e_{a_2} = V_t - (R_a + R_x)\, i_a'$

$= 240 - (0.2 + 2.325)(20) = 189.5$ V

$$\frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{1000} = \frac{189.5}{232} \Rightarrow \boxed{n_2 = 803 \text{ rpm}}$$



$$\left.\begin{aligned} e_{a_1} &= 232 \text{ V} \\ e_{a_2} &= 232 - 40 R_x \end{aligned}\right\} \Rightarrow \frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{n_1} = \frac{600}{1000} = \frac{232 - 40 R_x}{232}$$

solve for $R_x \Rightarrow \boxed{R_x = 2.325\ \Omega}$

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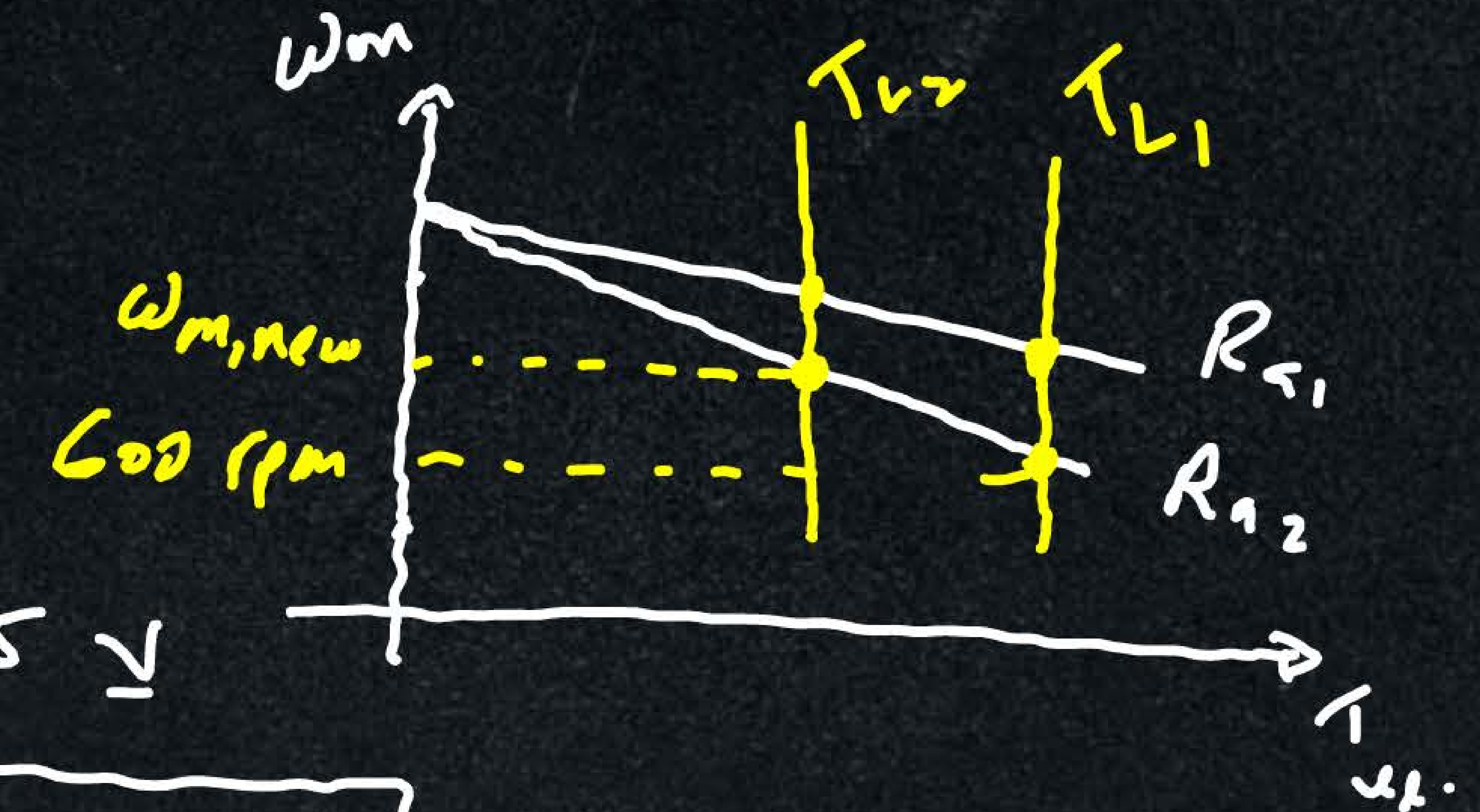
$e_{a_1} = 232 \text{ V}$



$$e_{a_2} = V_t - (R_a + R_x)\, i_a'$$

$$= 240 - (0.2 + 2.325)(20) = 189.5 \text{ V}$$

$$\frac{e_{a_2}}{e_{a_1}} = \frac{n_2}{1000} = \frac{189.5}{232} \Rightarrow \boxed{n_2 = 803 \text{ rpm}}$$

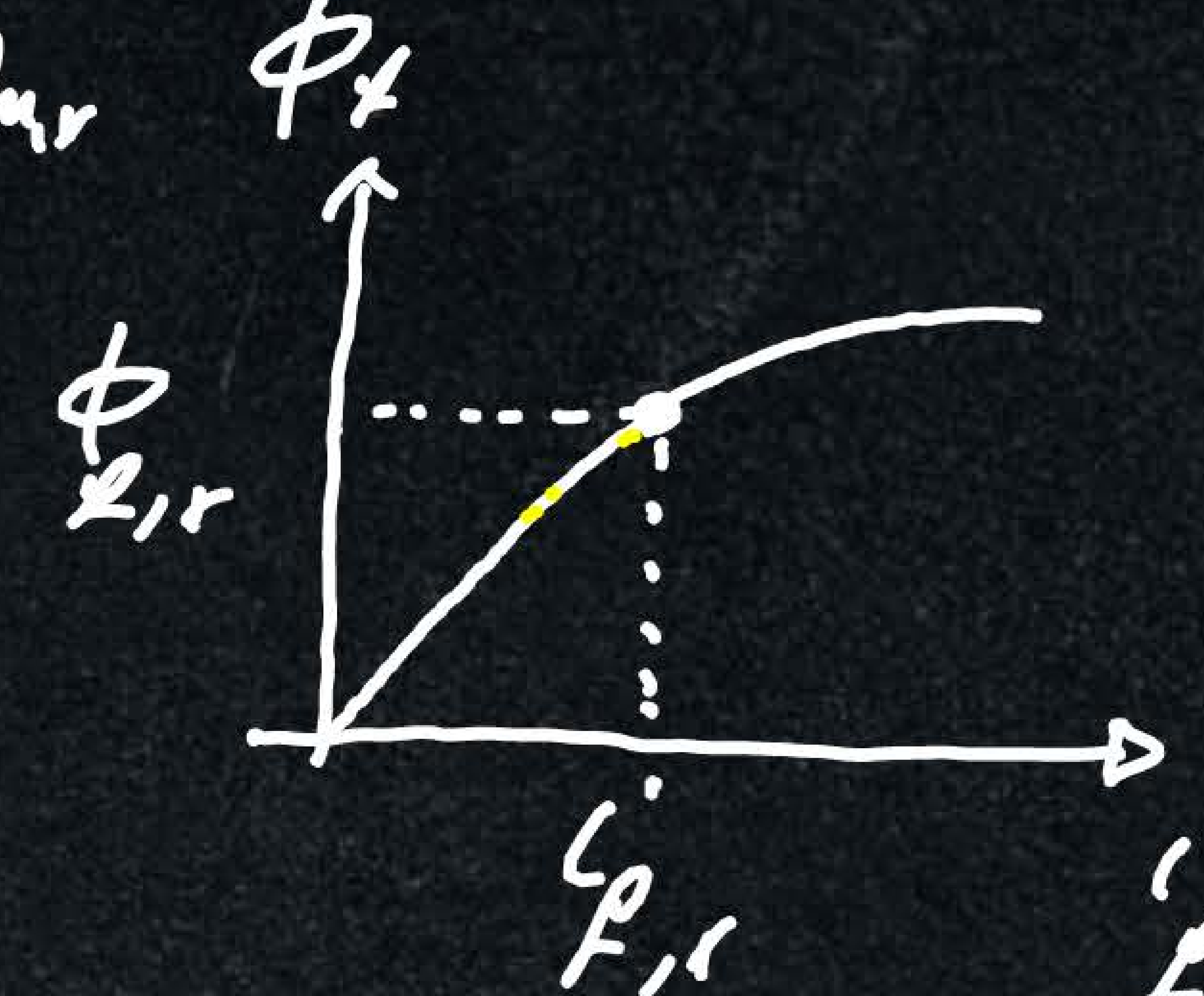
Principle of DC Machine drive

$$y = \frac{x}{z}$$

KVL in the armature circuit

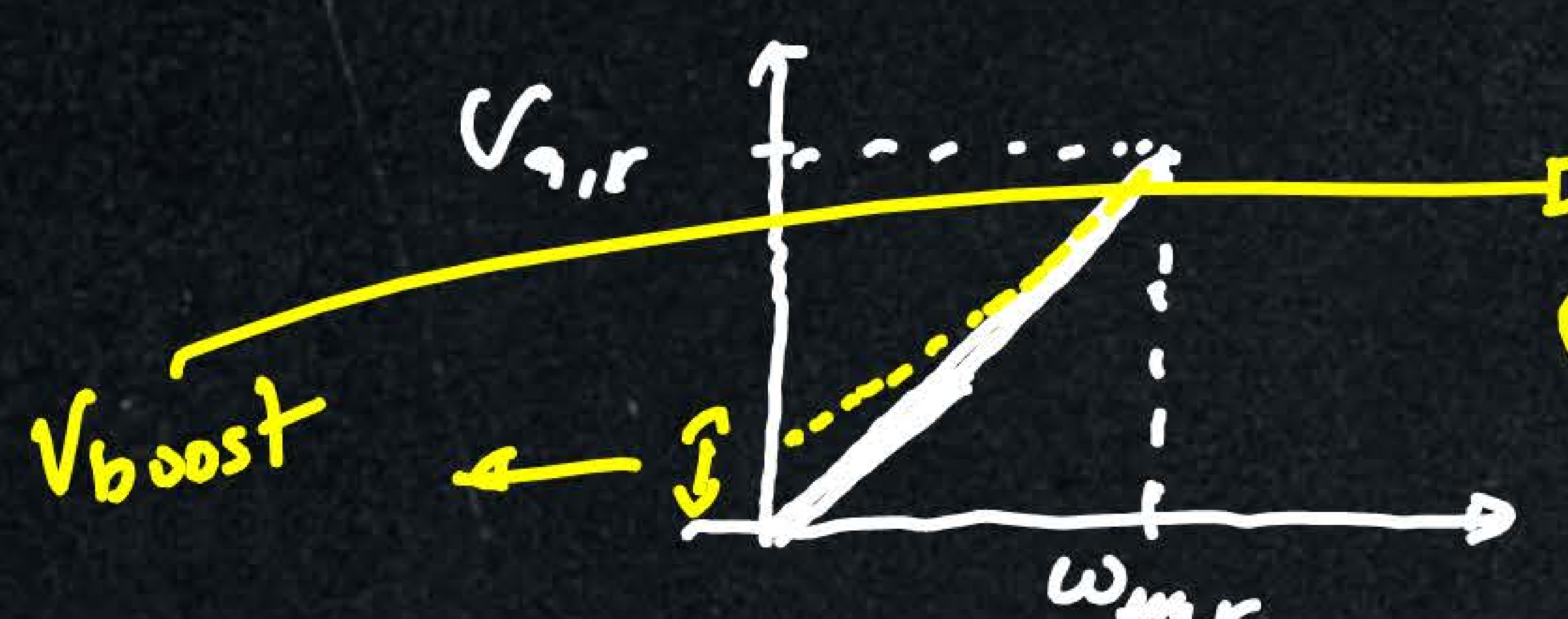
$$V_a = R_a i_a + K\Phi_f \omega_m \Rightarrow \omega_m = \frac{V_a - R_a i_a}{K\Phi_f}$$

<u>Armature control</u> : Ideal for speeds lower than ω_{mr}



$$\Phi_f = \Phi_{f,r} \Rightarrow \omega_m = \frac{V_a - R_a i_a}{K\Phi_{f,r}} \approx \frac{V_a}{K\Phi_{f,r}}$$

$$\Rightarrow \omega_m \propto V_a$$



to compensate the voltage drop across R_a.

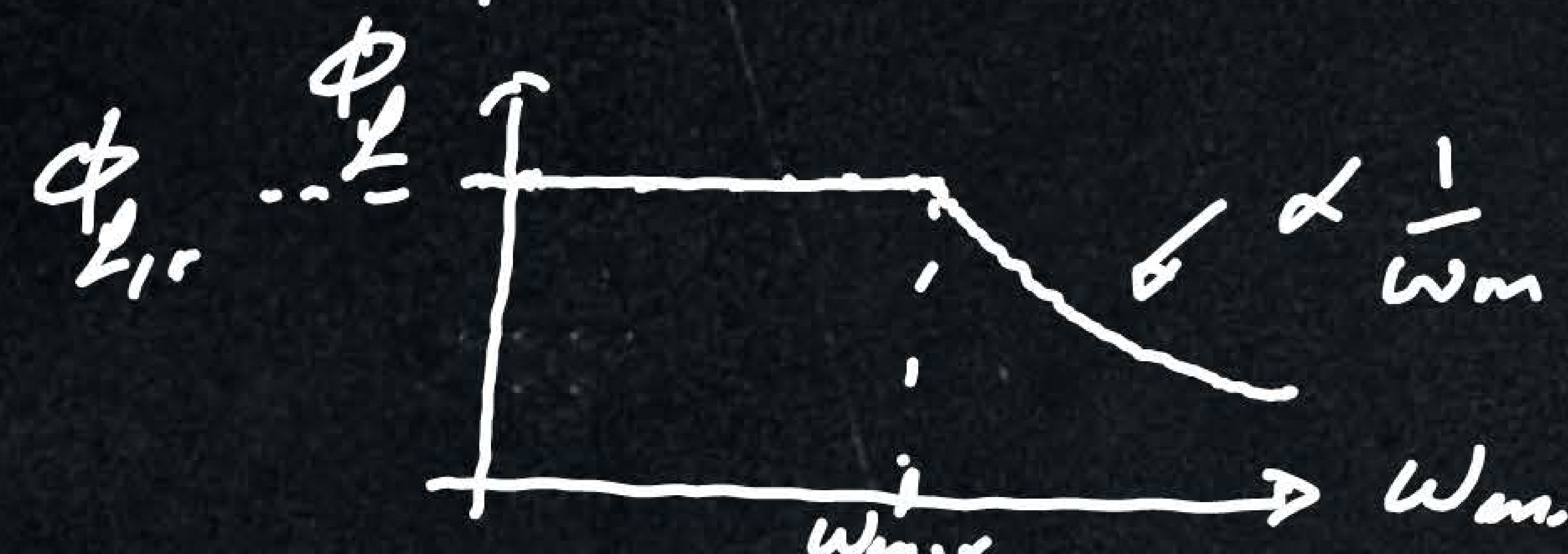
Field Control

$$V_a = V_{a,r} \Rightarrow \omega_m = \frac{V_{a,r}}{K\Phi_f} \Rightarrow \Phi_f \propto \frac{1}{\omega_m} \Rightarrow \Phi_f = \Phi_{f,r} \cdot \frac{\omega_{m,r}}{\omega_m}$$

$$\Phi_f \, \omega_m = A$$

$$\Phi_{f,r} \, \omega_{m,r} = \Phi_f \, \omega_m \Rightarrow \Phi_f = \Phi_{f,r} \frac{\omega_{m,r}}{\omega_m}$$

Ideal for speeds above than $\omega_{m,r}$



Armature and field control

Assume that $i_a = i_{a,r}$ = Rated current

$$\left.\begin{aligned}
&\omega_m \leq \omega_{m,r} \\
&\phi_f = \phi_{f,r} \\
&V_a \propto \omega_m \\
&T_{el} = K\phi_{f,r}\, i_{a,r} = \text{constant} = T_{el,r} \\
&P_a = T_{el}\,\omega_m = T_{el,r}\,\omega_m \Rightarrow P_a \propto \omega_m
\end{aligned}\right\}$$

Armature control mode

"Constant torque region"

$$\omega_m \geq \omega_{m,r}$$

$$V_a = V_{a,r}$$

$$\phi_f = \phi_{f,r} \frac{\omega_{m,r}}{\omega_m}$$

$$T_{el} = K \phi_f \, i_{a,r} = \frac{K \phi_{f,r} \omega_{m,r} \, i_{a,r}}{\omega_m}$$

$$= T_{el,r} \cdot \frac{\omega_{m,r}}{\omega_m} \Rightarrow T_{el} \propto \frac{1}{\omega_m}$$

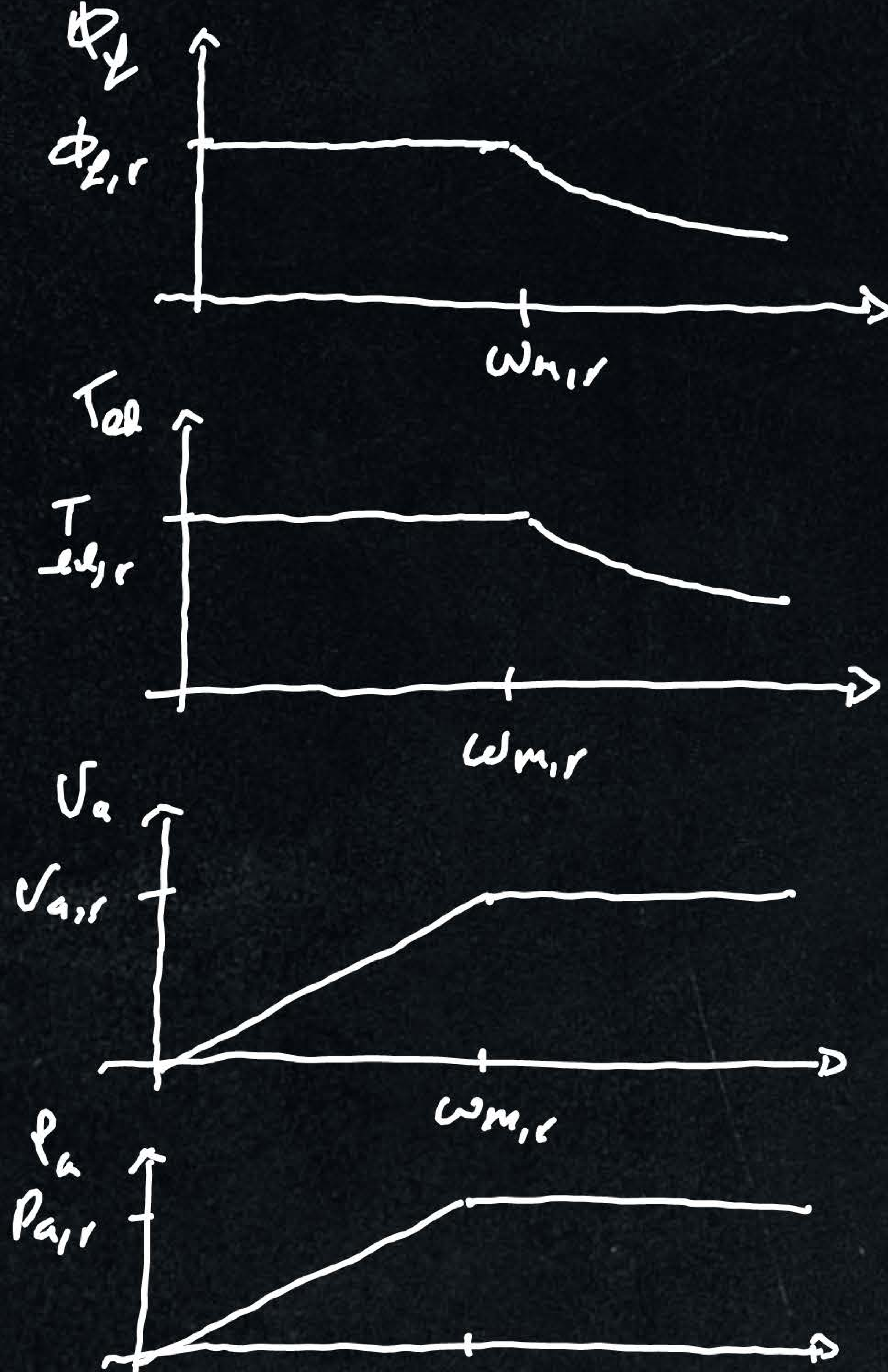
$$P_a = T_{el} \omega_m = T_{el,r} \frac{\omega_{m,r}}{\cancel{\omega_m}} \cdot \cancel{\omega_m}$$

$$= T_{el,r} \, \omega_{m,r} = P_{a,r}$$

2 kW

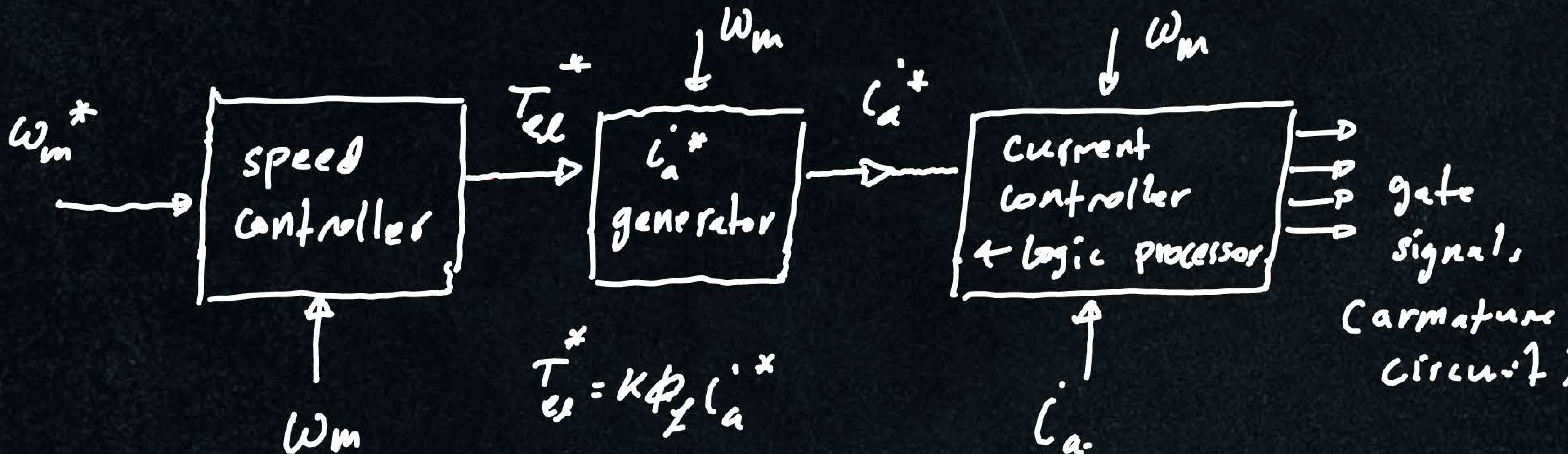
Field control mode

Constant power region!

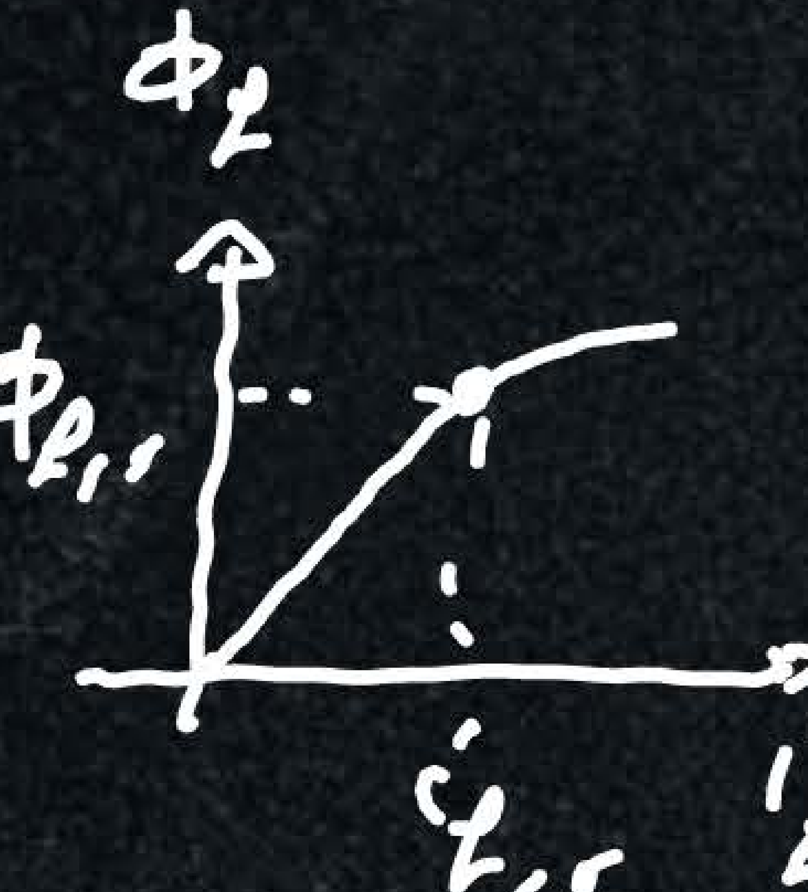

Φ_f
$\Phi_{f,r}$
$\omega_{m,r}$
T_{el}
$T_{el,r}$
$\omega_{m,r}$
V_a
$V_{a,r}$
$\omega_{m,r}$
P_a
$P_{a,r}$

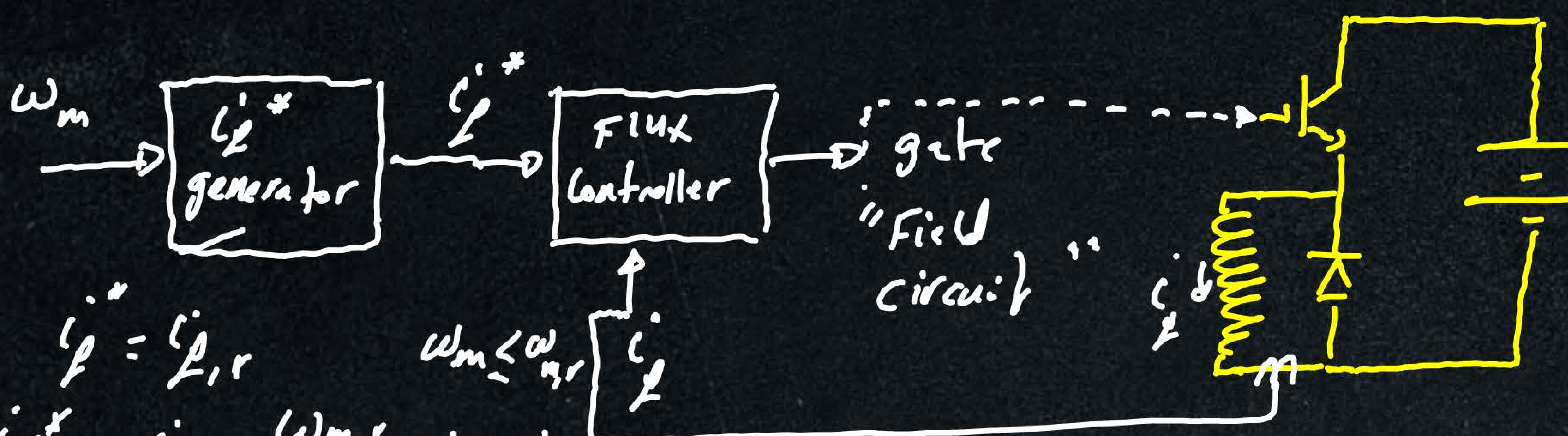
Control system of DC Machine Drive

speed controller
current controller
Flux controller
* → reference



$T_{el}^* = K\Phi_f i_a^*$



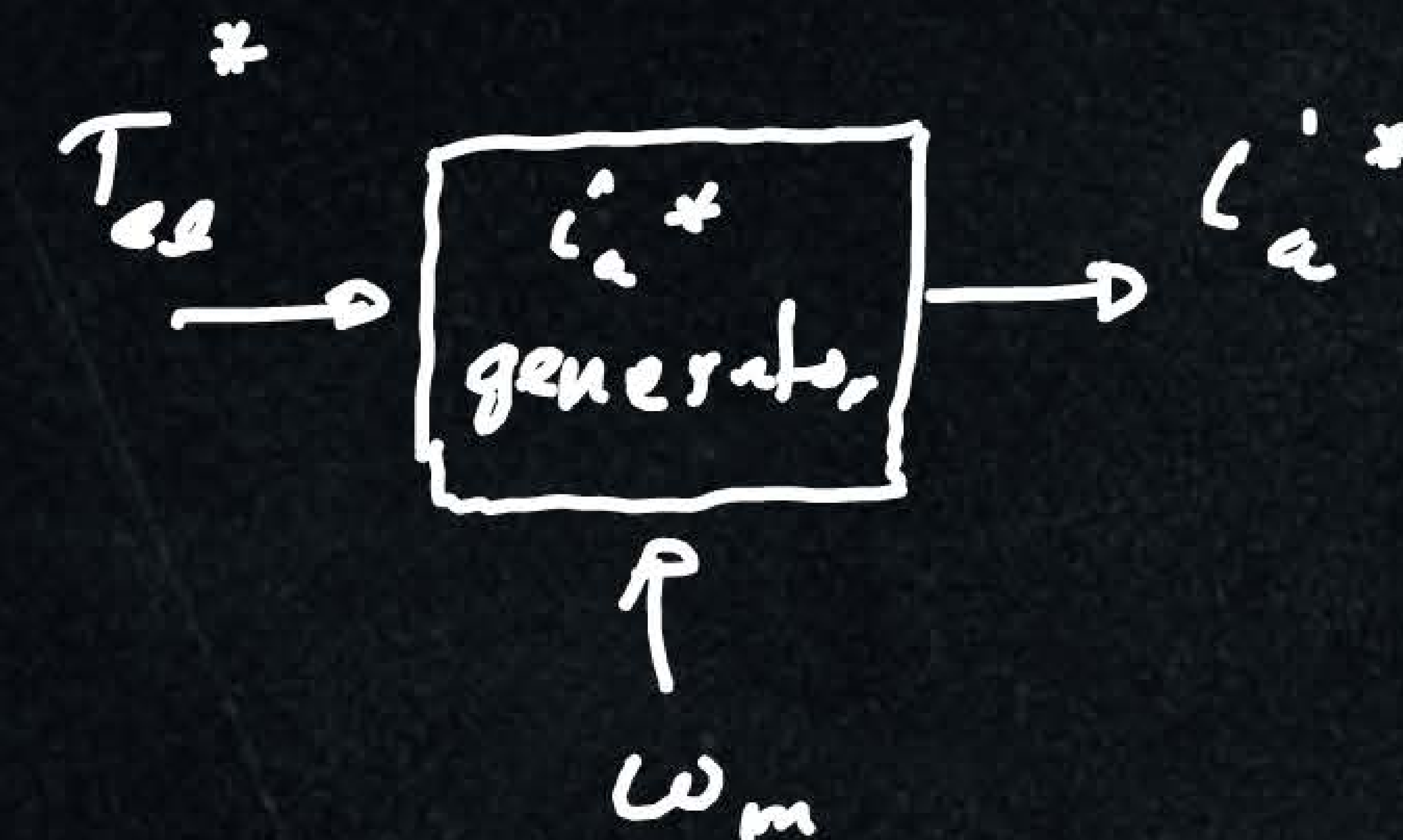


$i_f^* = i_{f,r}$ $\quad \omega_m \le \omega_{m,r}$

$i_f^* = i_{f,r}\dfrac{\omega_{m,r}}{\omega_m}$ $\quad \omega_m \ge \omega_{m,r}$

i_a^* Generator

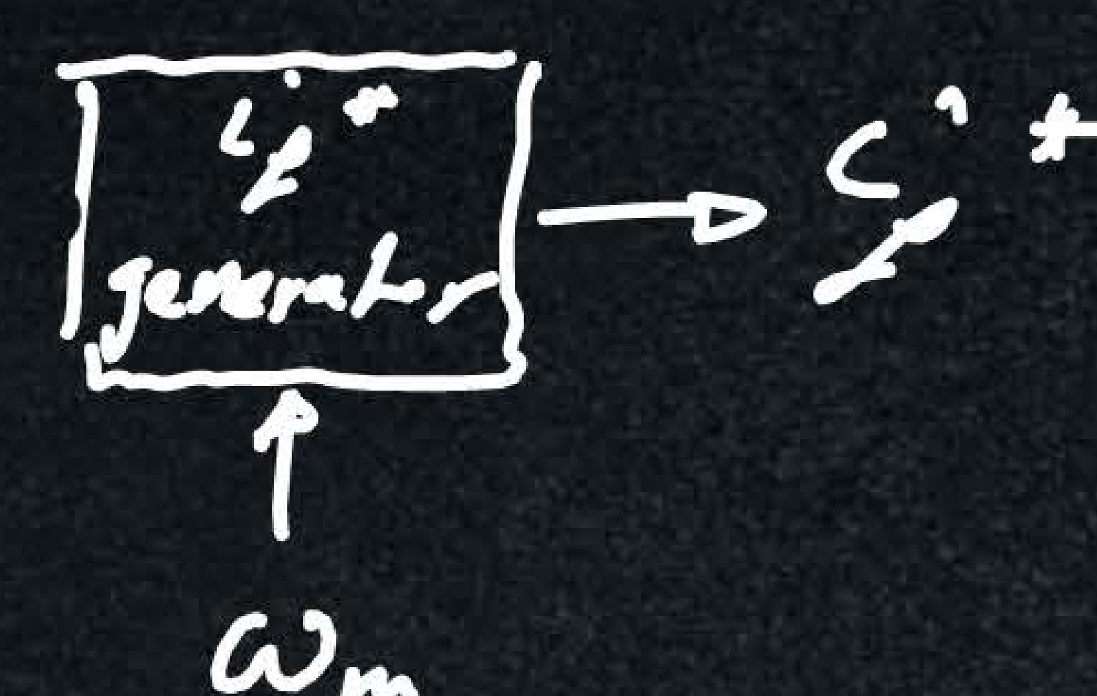
$$T_{el} = K\Phi_f \, i_a$$



$$\omega_m \le \omega_{m,r} \rightarrow i_a^* = \frac{T_{el}^*}{K\Phi_{f,r}}$$

$$\omega_m \geq \omega_{m,r} \rightarrow i_a^* = \frac{T_{el}\,\omega_m}{K\Phi_{f,r}\,\omega_{m,r}}.$$

i_f^* Generator



$$\omega_m \le \omega_{m,r} \Rightarrow i_f^* = i_{f,r}$$

$$\omega_m \geq \omega_{m,r} \Rightarrow i_f^* = i_{f,r}\,\frac{\omega_{m,r}}{\omega_m}.$$

Speed Controller

It is designed using Newton's 2^{nd} Law :-

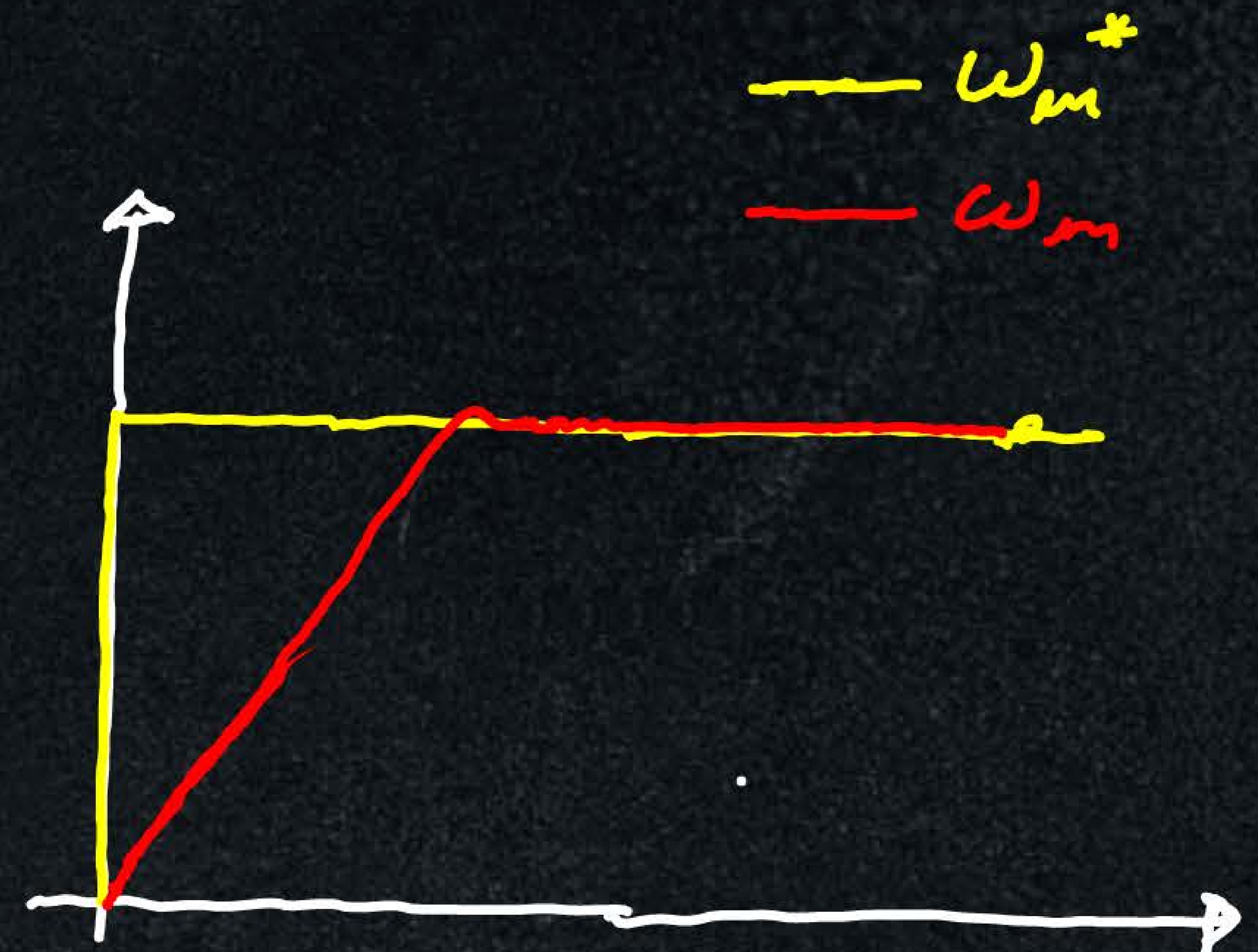
$$T_{e} - T_{l} = J \frac{d\omega_m}{dt}$$

$$\omega_m^* \longrightarrow \bigcirc_{-}^{+} \xrightarrow{e} \boxed{K_1 e + K_2 \int e\,dt} \longrightarrow T_{e}^* /$$

ω_m

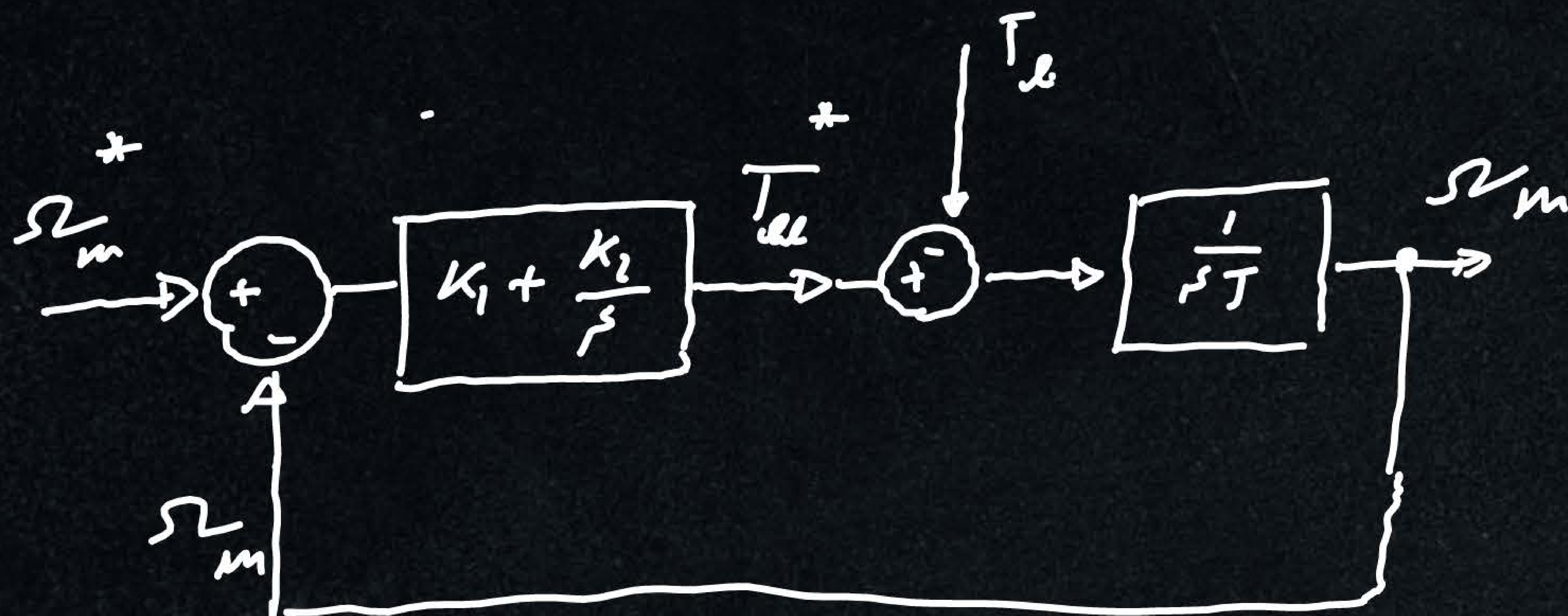
PI-controller

time-domain



Design of speed controller in s-domain

$$T_{el} = T_l + J\frac{d\omega_m}{dt} \xrightarrow{\mathcal{L}} T_{el} = T_l + sJ\,\Omega_m$$

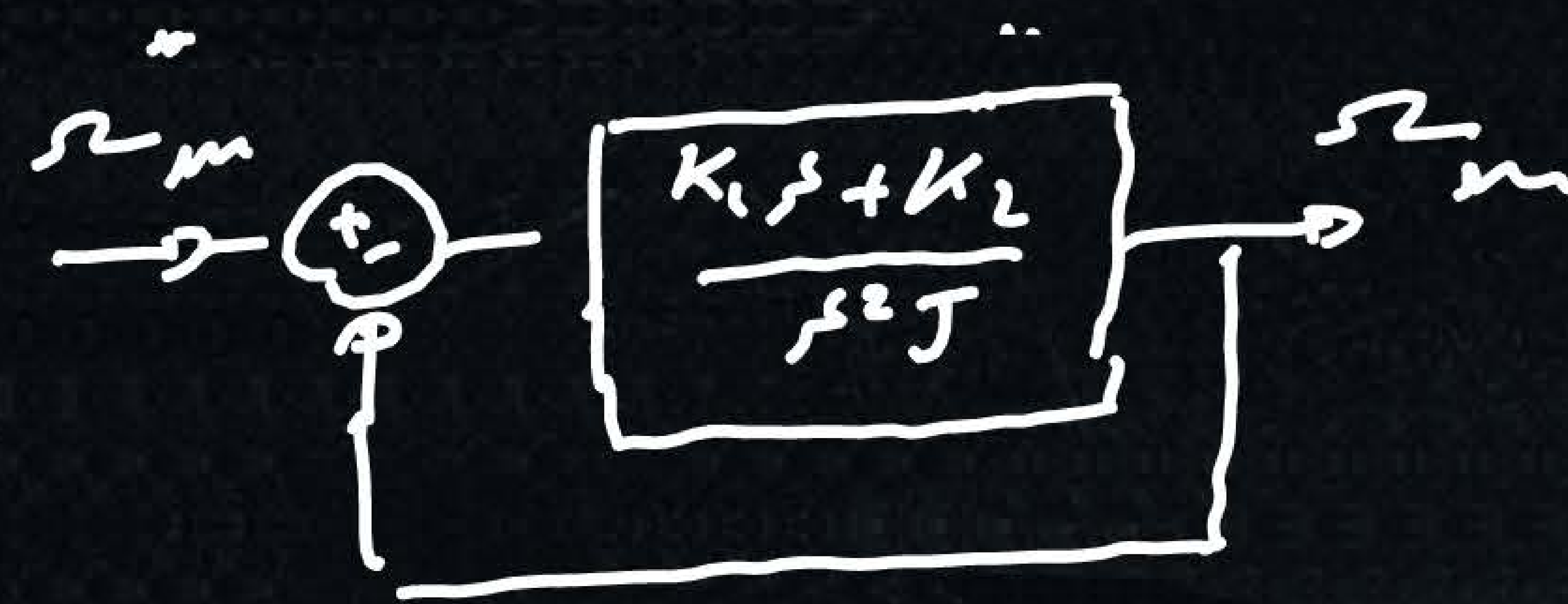


Linear closed loop control system

$$\Omega_m = T_1\,\Omega_m^* + T_2\,T_l$$

$$\omega_{m,ss} = \omega_m(t\to\infty) = \omega_m^* \;\;??$$

$T_1 : T_l = 0$



$$T_1 = \left.\frac{\Omega_m}{\Omega_m^*}\right|_{T_l = 0} = \frac{G}{1 + GH} \quad ; \quad G = \frac{K_1 s + K_2}{s^2 J}$$

$$H = 1$$

$$T_1 = \frac{\frac{K_1 s + K_2}{s^2 J}}{1 + \frac{K_1 s + K_2}{s^2 J}} = \frac{K_1 s + K_2}{s^2 J + K_1 s + K_2} = \frac{K_1}{J}\left[\frac{s + K_2/K_1}{\underbrace{s^2 + \frac{K_1}{J} s + \frac{K_2}{J}}_{s^2 + 2\zeta \omega_n s + \omega_n^2}}\right]$$

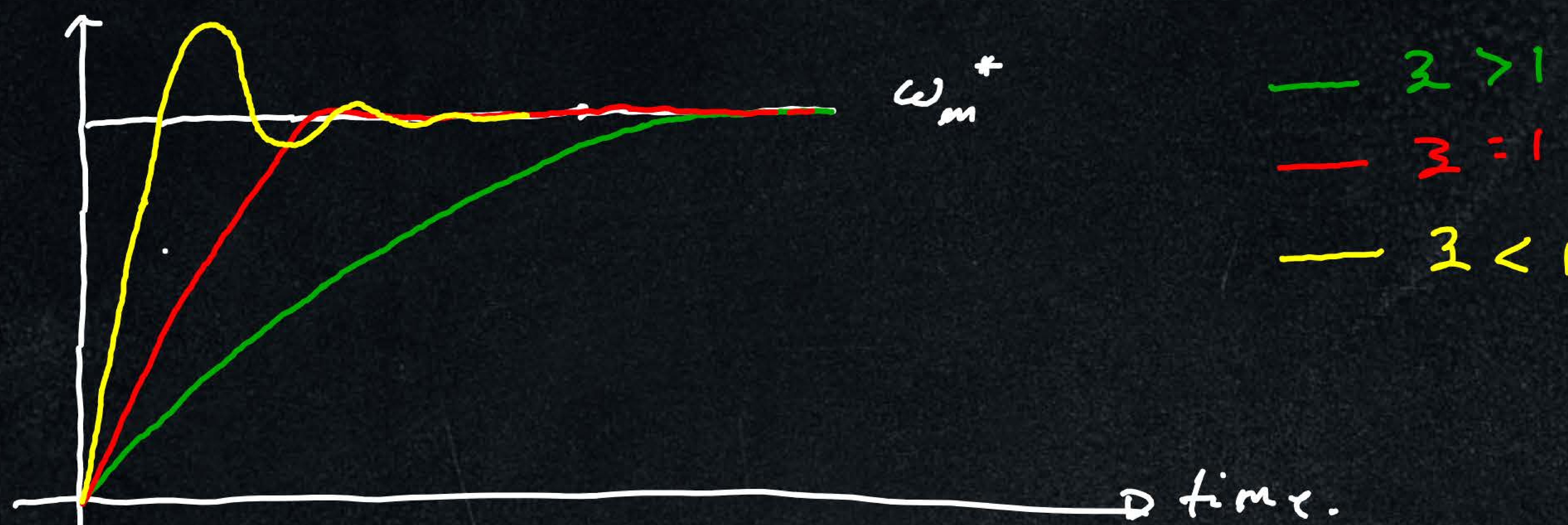
ζ: damping ration

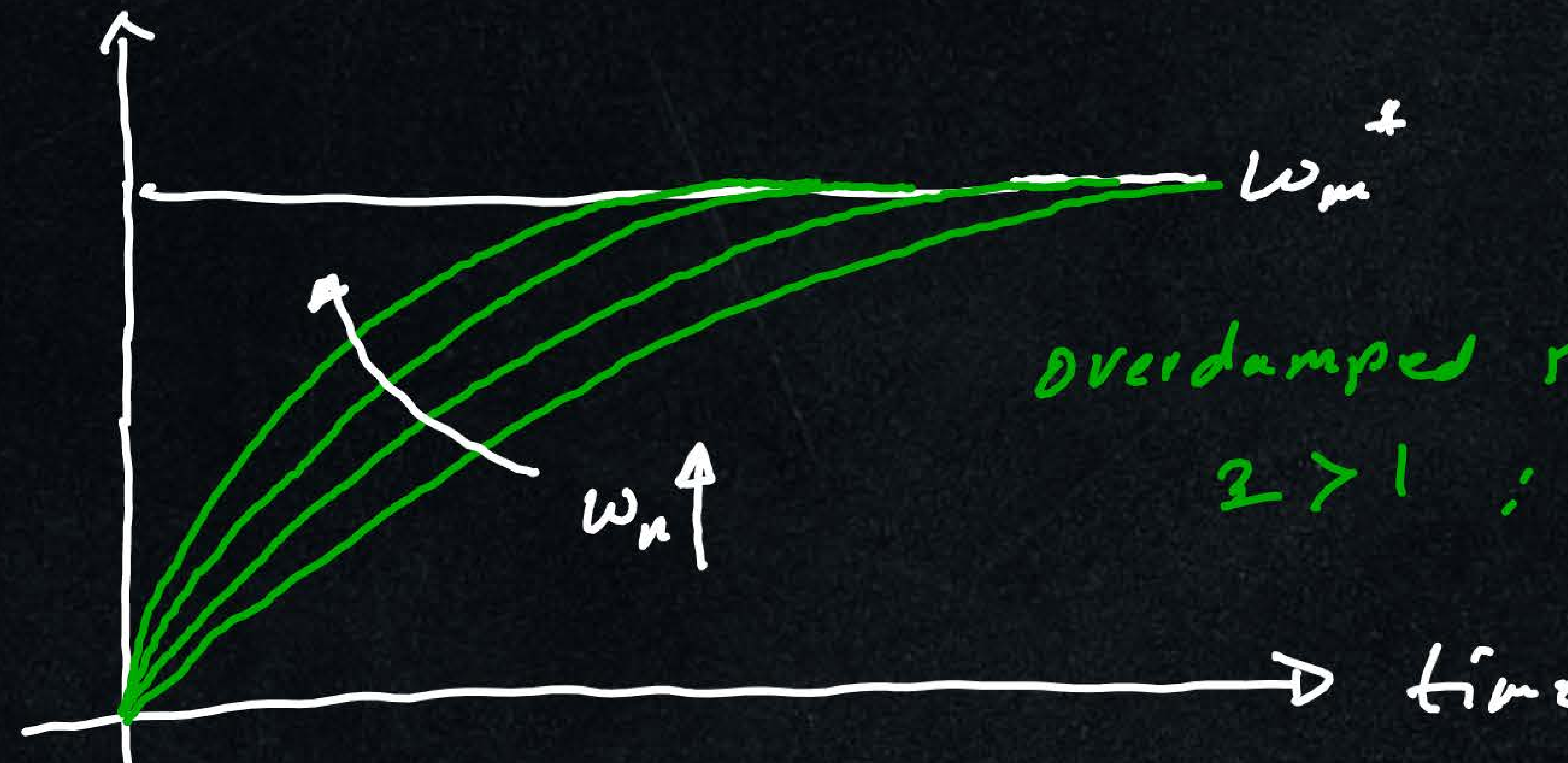
ω_n: Natural frequency

$\zeta < 1$ Under-damped

$\zeta = 1$ critically-damped

$\zeta > 1$ over-damped



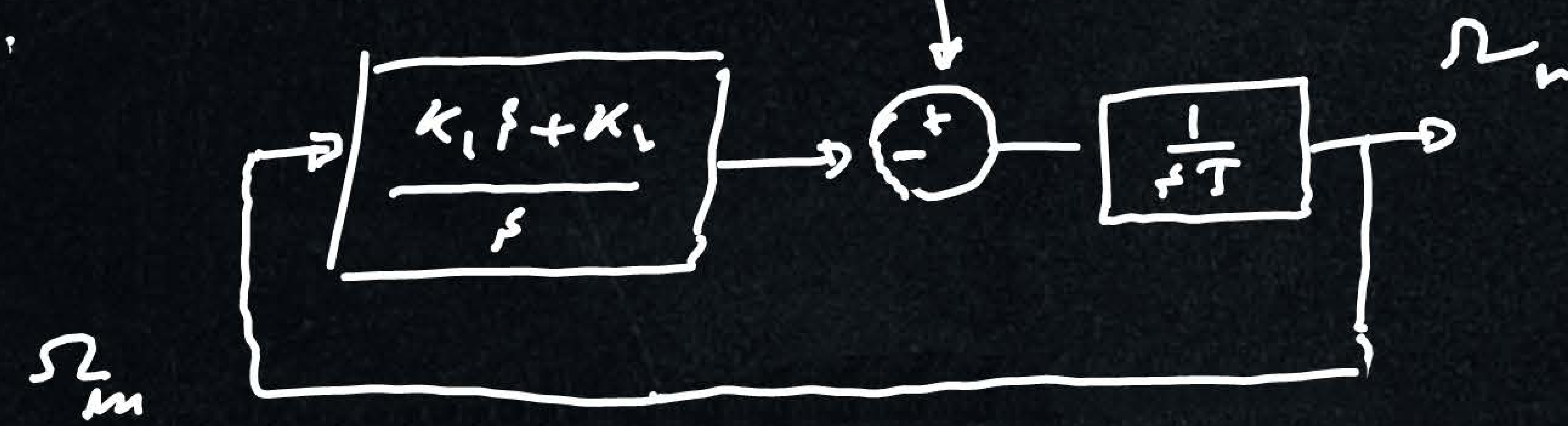


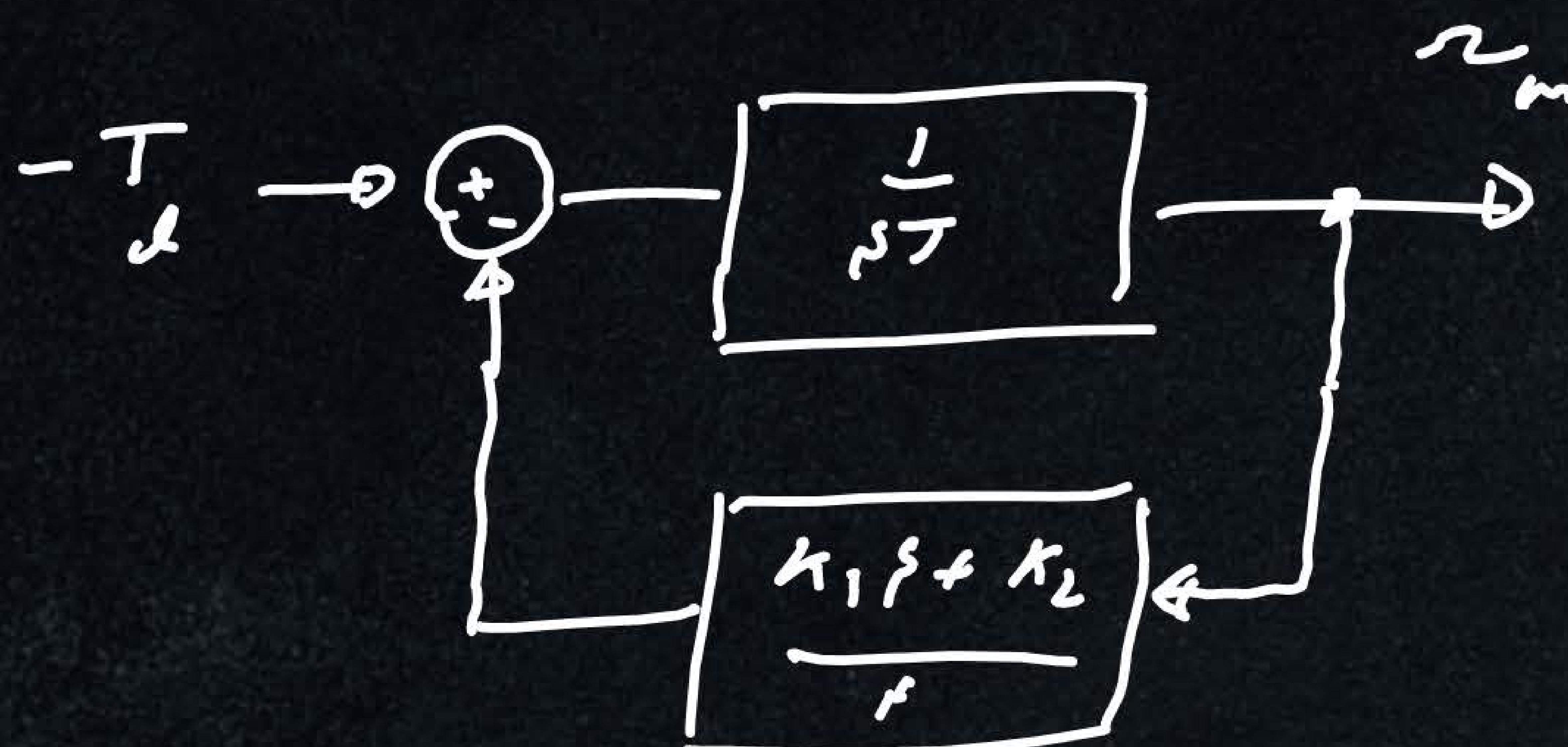
overdamped response

$\zeta > 1$; $\underline{\zeta = 5}$

$$T_2 : \Omega_m^* = 0 \; ; \quad T_2 = \frac{\Omega_m}{T_d}\bigg|_{\Omega_m^* = 0}$$

$-T_d$





$$T_2 = \frac{\Omega_m}{T_d} = \frac{-G}{1 + GH}$$

$$G = \frac{1}{sT} \; , \quad H = \frac{K_1 s + K_2}{s}$$

$$T_2 = -\left[\frac{\frac{1}{sT}}{1 + \frac{K_1 s + K_2}{s^2 T}} \right] = \frac{-s}{s^2 T + K_1 s + K_2} = -\frac{1}{T} \, \frac{s}{s^2 + \frac{K_1}{T} s + \frac{K_2}{T}}$$